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Designing a Standards- Based Math Curriculum

KNOWLEDGE ■

Education is preparation for life. Derived from the Latin, it also means *to lead forth*—perhaps to knowledge. But what is the knowledge to which we educators must lead our students? Knowledge is defined as an acquaintance with a fact, a perception, or an idea. A suggested classification of knowledge divides it into procedural and conceptual components. The two categories are distinguishable and yet intersecting. They are not hierarchical; one does not necessarily come before the other. They differ in that procedural knowledge is more rigid and limited in its adaptability, but highly efficient, especially when it is applied with meaning. Conceptual knowledge is more flexible—it reorganizes and stretches itself as it tries to connect new perceptions and previous generalizations. Conceptual knowledge may then reform itself as a new generalization. Reasoning requires conceptual knowledge (for an in-depth discussion of conceptual and procedural knowledge, see Hiebert & Wearne, 1986).

A chef following an often-used recipe is efficiently carrying out his procedural knowledge. He knows that it must be done in a certain order and with specific ingredients and quantities. Suppose one of his ingredients is unavailable. He has a problem. When he tries to innovate with a substitute ingredient, he calls upon his conceptual knowledge, reorganizes it; connects it to other concepts and perhaps to a new generalization; and then with practice connects it to a new procedure. The best procedures are those built with conceptual knowledge, those learned with meaning. Conceptual knowledge may, however, also come from procedural knowledge. An infant puts blocks one on top of the other, perhaps in a self-initiated procedure or perhaps copying an adult. Eventually a concept is formed: The larger blocks need to be at the bottom. *The conceptual and procedural knowledge components that we expect our students to have,*

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and that therefore we must lead them to, form our content standards. Our expectations are based on our own knowledge, our experience, and our predictions about what our students will need. Content standards describe what we value and what we want our students to know or be able to do.

The reason for the discussion above is that in the past mathematics has often been taught as a set of specific procedures, sometimes disconnected from the real problems that will confront us in life, and frequently without clarification of the mathematical concepts that are embedded within the procedures. Doing mathematics requires a set of clarified concepts and procedures that develop over time. Experiences with objects and verbalization—both monologic and interactive—help that development. The concepts enlarge our capability to solve problems; the procedures make us more efficient. An understanding of this dual nature of knowledge also provides a rationale for the organization of standards-based curriculum.

■ ORGANIZATION AND DESIGN OF CURRICULA

Although concepts and procedures develop individually for each student over time, it is useful for the teacher to know what the necessary ones are—the ones that can help the student do mathematics and solve mathematical problems. The common procedures we use and some of the concepts we share are in what has been called “*the consensual domain*” (Cobb, 1990). This shared knowledge is the content of our curriculum. The framework for this curriculum content includes normed expectations for achievement or **content standards**. The multiplication facts are shared knowledge in the consensual domain. That we expect third and fourth graders to know their multiplication facts is a normed expectation or standard based on history, teachers’ experience, and the average achievements of children in these grades. In the present societal context, the standards are also formally set and monitored by state and federal guidelines and legislation.

Content standards organize and describe the curriculum. They serve as guides for instruction that are planned to help students achieve the knowledge of the consensual domain. They tell us what students should know or be able to do. Many state-developed documents label their content standards as **performance indicators** that emphasize what students should be able to do. Performance indicators that focus primarily on procedural knowledge are then used as a basis for matching measurement guidelines or **test expectations**, which are translated into the mandated high-stakes assessments that are used to hold schools accountable for the performance of their students (Solomon, 2002, 2003). **Embedded concept** knowledge, which describes what students should know, is assumed necessary for the measured performance of procedural knowledge, but the concepts are rarely stated explicitly. Clearly stated mathematical concepts within curriculum documents may prove to be helpful in achieving consensus and guiding instruction. Moreover, it is important not to neglect separate measures of the embedded concepts. Test items that ask for explanations specifically seek concept knowledge and can be used as a diagnostic that determines why a procedure is not understood.

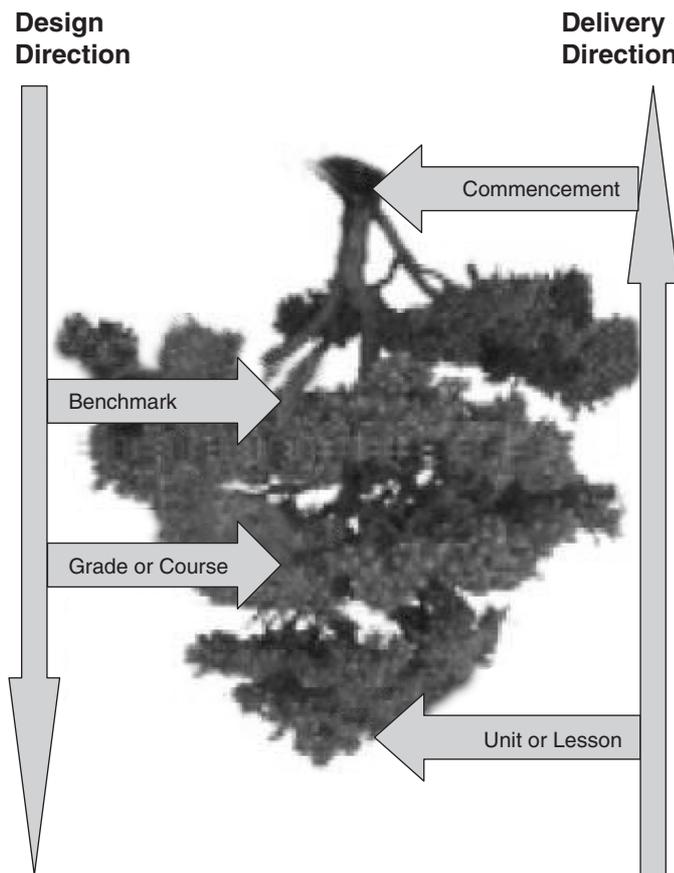


Figure 1.1
Curriculum Planning

When constructed, classroom activities and assessments should be reflections of the concepts and procedures of the standards. An analogy that might help is to compare them to the two sides of your hand. The back of your hand, like the standard, defines its form and its potential, but the palm is the implement and measure of what your hand does. Content standards define the framework for the actions of instruction and the assessments we need to help guide us and our students while providing accountability to our publics.

Standards can be very general statements of expectations at a terminal or commencement point or more specific and assigned to a particular stage in development or grade level. The upside-down tree in Figure 1.1 illustrates a design process for standards-based curricula (Solomon, 2003). Like the trunk of a tree, general standards lead to a widely reaching set of more specific branches, twigs, and leaves. Curriculum is *designed down* from more general *commencement* levels to the more specific *benchmarks* and then to the even more specific levels of the course, grade, and unit. But it must work both ways. Just as the leaves of a tree must manufacture food and nurture the trunk, the more specific “designed-down” content standards of every lesson must feed the general ones—they make the general ones happen. Curriculum is *delivered up*—up toward the general or commencement standards. None of this works if the connections of internal flow are impeded. The junctures where twigs meet branches and branches meet trunks are particularly important. The outcome of each lesson of the leaves is fed

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through a twig to the branch that is the unit and then into a larger one that is the grade level. Several grade levels may feed into a larger branch at a benchmark juncture and this, in turn, finally meets the main trunk. The tree is shown upside down because the design is the beginning and we think of the processes as “design down” and “deliver up.” At the same time, there must be horizontal articulation. As the leaves turn toward the sun, the carbon dioxide must enter them. There must be a balance between the concepts and the procedures.

The preplanned design is only the first step. The settings and activities of well-planned classroom activities must have a reasonable probability of helping *all* students to be successful in these measures. They should encompass a wider scope of the variables of the classroom experience: the teachers’ knowledge and carefully reviewed previous experience, the discourse, the materials, the allocation of time and space, the cultural and social contexts of peers and adults.

■ THE MATHEMATICS CONTENT STANDARDS: KEY IDEAS¹

What kind of mathematical knowledge do we expect of all students when they enter the technological world of the third millennium? What are the steps for getting them there? Consider the upside-down tree for mathematics curricula. Beginning at the trunk, at the commencement level, we should expect that *all* students can do the processes of mathematics, such as reasoning, communicating, and problem solving. Nevertheless, the processes of mathematics are not performed in a vacuum. They depend upon and produce a content set of conceptual and procedural knowledge about mathematics. However, before we address the specifics of the processes and content knowledge standards, there are some key ideas that should be considered.

- In its traditional sense, mathematical **reasoning** includes both quantitative and spatial concepts, but it also has embedded *verbal constructs and a special language*. In addition, effective reasoning may also involve *metacognitive processes*. Thinking about what you are doing and purposely comparing problems and solutions may increase the power of reasoning (Kramarski & Mevarech, 2003).

- The special language of mathematical **communication** involves a system of **numbers and other symbols**. The symbols represent values and orders or something that changes the value. Not only do we need to use this language to communicate with others, the symbols may be necessary for our own internal concept formation. There is also a special language for sharing proof.

- A logical search for truth or **proof** requires reasoning and is a special power of mathematics (Herbst, 2002). Proof to oneself also strengthens the constructions of knowledge.

- As we observe, reason, connect, and communicate, we can develop and use an intuitive **number and spatial sense** that allows us to estimate values, judge relative size, visualize hidden parts of forms, decide on appropriate

strategies for problem solving, predict the result of operations and transformations, and evaluate the reasonableness of our problem solutions.

MATHEMATICS CONTENT STANDARDS: ■ PROCESSES AND DISPOSITIONS

Our expectations of students' ability to do the processes of mathematics reflect the way research has shown us that all learning happens; like all learning, doing mathematics involves connecting prior knowledge and new perceptions. Doing mathematics requires and builds both conceptual and procedural knowledge. Doing the processes of mathematics means that students can do the following:

- *Perceive* and make observations of the world from a mathematical perspective, sensitive to similarities, differences, patterns and change in size, value, time, and form.
- *Connect* these observations to each other and to other concurrent observations and prior knowledge (e.g., the form of a sphere and a rolling ball).
- *Represent* forms and number systems in multiple ways and models to help them visualize, communicate ideas, organize data, and construct concepts.
- *Communicate* what they perceive to others using multiple forms of representation and the special language of mathematics.
- *Analyze and solve problems* using mathematical *reasoning*, which is based on conceptual knowledge, and do this efficiently with meaningful procedures.
- *Justify* and *defend* their solutions with logical *proofs*.

Conceptual knowledge can also be knowledge about oneself; it can be an attitude, a value, or a goal (Anderson & Douglass, 2001). Attitudes, values, and goals control the learning process. Doing mathematics also requires that students:

- Have *confidence* in their ability to do mathematics.
- *Appreciate* the beauty and power of mathematics.

MATHEMATICS CONTENT STANDARDS: ■ THE KNOWLEDGE CONTENT SET

For the purpose of description, we can organize the knowledge content set into six major branches. It is important to realize, however, that these branches are overlapping—both in their interdependence and in their function as we enact mathematical processes. For example, our operations are dependent on our number system, and our number system determines the form of our operations. We need knowledge of our number system, measurement, and data representation as we communicate to others what we have perceived. The content

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standards described in Chapter 2 include designed-down concepts and procedures from the following major commencement level branches:

- **Number system:** The language of our common *number system* (which is based on our genetically and experientially determined sense of space and quantity and the number of finger or toe digits) allows us to perceive and communicate quantities in words and symbols. By making the left-to-right position of the symbols have different values we are able to express all quantities with only ten symbols including the placeholder zero. There are other number systems.

- **Operations on numbers:** We can perform *operations on numbers*. Operations are systems that help us solve problems that involve change or comparisons. They allow us to determine values not directly counted or measured. Reasoning with our conceptual knowledge and using our number sense can help us predict the result of operations. The language of real-world problems needs to be translated into the language of mathematics so that we can solve the problems efficiently by performing operations.

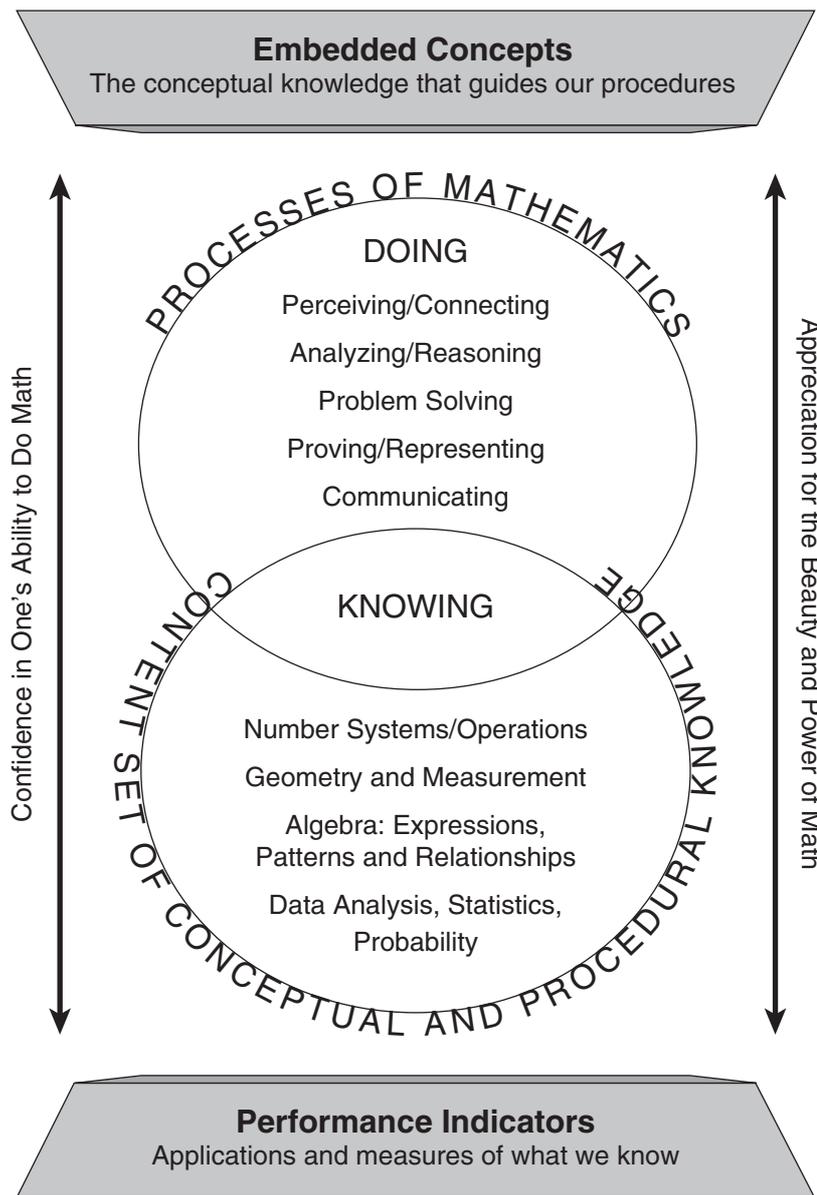
- **Geometric forms and properties:** Defined two-dimensional surface areas and three-dimensional objects that take up space have different *geometric forms and properties*. Knowledge of the dimensions and properties of these forms, and the relationships among them, helps us solve problems and make use of the systematic relationship between the types of forms and their practical functions (e.g., the rolling sphere, the Roman arch, the sturdy triangle).

- **Measurement and data collection:** We use our number system to *measure* the dimensions and characteristics of objects and areas as they exist, or change in time and space. We also measure time itself and other values and phenomena such as money, light, wind, energy, votes, and the popularity of TV shows. Collections of *measurements* are called *data*.

- **Algebra: Patterns, expressions, relationships, and functions:** Within the systems of numbers, forms, and data there are recognizable *patterns* and *relationships*. Patterns help us reason, organize, and *automatize* concepts (see ahead) into more efficient procedures. We use *symbols* to express the patterns and relationships. The symbols can represent either variable or constant values. When patterns express specific relationships between constant and variable values, they are called *mathematical functions* (each input has a specific output or rule that guides it). Conceptual and procedural knowledge of functions is very useful in complex problems solving and prediction.

- **Data analysis, statistics, and probability:** Data can be collected and analyzed to show patterns and trends that can help us make predictions. Statistics are systems used to organize data and analyze it in many different ways. Some events are clearly predictable, but others are *uncertain*. *Probability* systems help us deal with uncertainty by giving us a way to have reasonable expectations about the possibility of the occurrence of an event.

Figure 1.2 represents an organization of the intersecting sets of process and content branches of mathematics as well as the dispositions or attitudes that affect all of them. They are placed between the inclusive and articulated

**Figure 1.2**

The Content Standards:
What Students Should Know
and Be Able to Do

anchors of Embedded Concepts and Performance Indicators. Our distinction between these two anchors is that the embedded concepts focus clearly on the underlying ideas of mathematics that we wish our students to know, while the performance indicators focus on the demonstrations, applications, and measures of that knowledge.

The presentation of the very specific designed-down content standards in Chapters 2 and 3 incorporates several critical premises about what students need, how learning happens, and how teachers use curriculum. The content and organization of these chapters responds to these premises in the specific manner described below.

Premise 1: Inclusivity. Although Chapters 2 and 3 represent most of the general topics typically included in math standards for Grades K–5, they do not pretend to be all-inclusive, neither of the general topics

nor of the embedded concepts and procedures within them. They may, however, be a substantive starting place for designing and implementing curriculum and assessments. Teachers and their students may discover needed additions and make some corrections. The important thing is to recognize the ideas for oneself, communicate them to others, and then reach a useful consensus about what is critical for us all to learn.

Premise 2: Timing. For some concepts and for some students learning happens all at once. For many others it is an iterative process that takes place over time as students develop meaning in a very individualized way. Sometimes this meaning is “buggy” or incorrect and is corrected by new perceptions. The grade-level expectations in Chapter 2 are therefore presented in three phases: exploration, concept mastery, and procedural or algorithmic mastery. In some cases, this sequence may all happen in one grade—even in one lesson; in others the span may be longer than three grades. The expectations listed are suggested medians based on observations of students and references to varied texts and assessments. Teachers should adjust these on a local basis. The idea behind the three phases is that as students engage in the mathematical processes they begin with explorations: perceiving, observing, trying to find solutions. At first, they may find solutions without crystallizing a concept that is permanently implanted as a schema in memory. They may need help from teachers and/or peers in the form of interactive dialogue to do this.

Premise 3: Algorithms. Once the concept is formed, further experience may *automatize* its retrieval from memory, and learners can incorporate it into strategies or procedures that can be efficiently employed. The common algorithms are an example of procedural strategies. The algorithms were invented over time as efficient procedures for solving common mathematics problems. Students should be able to use the algorithms in the consensual domain but be encouraged to invent and prove their own strategies as well. A good rule of thumb for the use of traditional algorithms by students is to evaluate the potential usefulness of the algorithm—as a tool for solving real problems in the current technological world; as a written record that might help organize concepts; and as a procedure, learned in its application to simple, easily understood problems, that can then be extrapolated to more complex applications. In the past, much time has been spent by students in the process of developing skill, speed, and accuracy in using these algorithms—perhaps detracting from a focus on the more powerful ideas of mathematics and distracting students from interpretations of problems that would allow for quick mental solutions. These algorithms were most often learned without attention to understanding how and why they worked. Analysis of problems hinged on *key words* that told you which algorithm to use; the selected procedure was applied without conceptual understanding or recognition that sometimes the problem could be easily solved mentally.

Students should be encouraged to use their concept-based number or spatial sense to interpret a problem and estimate its answer before applying a procedure. They may, however, need the teacher's help and some practice to reach the third procedural mastery phase. In some cases, when the child's struggle with the construction of a concept is discouraging, it may even be necessary to move over the concept to a rote learned procedure, but attempts to recursively revive the concept should continue. In general, moving to a procedure first should be avoided because there is some evidence that learning a procedure rote—without the underlying concepts—may encumber concept development and handicap further development of mathematical processes (Morrow, 1998; Usikin, 1998).

Premise 4: Automaticity. In order to be able to estimate and use reasoning to solve problems, students need a repertoire of easily retrieved bits of conceptual and procedural knowledge. That repertoire includes related addition and subtraction facts for combinations up to 20 and multiplication and division tables to 12 as well as the standard unit equivalents for measurement. The need for automaticity may have been somewhat subsumed by the prevalent use of calculators, but a missing bank of automatized facts may be detrimental to the development of mathematical knowledge. My own experiences with children and with other cognitive research has also demonstrated that the earlier the requirement or motivation for automatization, the easier it is to embed facts in long-term memory—and retain them there. As it does for the learning of a second language, the brain may have optimum development times for automatization of number facts.

We use the term “automatize” as an outcome descriptor to differentiate from the traditional term *memorize* in order to emphasize that the process of imprinting the facts should be a meaningful one, utilizing reasoning and pattern recognition. Automaticity implies fast retrieval from memory, but strengthened by reasoning and the conceptual knowledge of patterns it also allows for fast reconstruction should a fact be temporarily lost (Cumming & Elkins, 1999; Phillips, 2003).

Premise 5: Verbalization and Language. Although I have tried to use clear and simplified language, the words of the content standards are adult terms that express the consensual domain. It is not necessary for the children to use the exact words, as long as the teacher is convinced that the meaning has been correctly constructed. In some cases, students will be able to demonstrate the concept only by doing things with objects or giving examples, but verbalization of the concept in different ways should be encouraged and listened to. Verbalization of a concept helps place the concept in long-term memory. In order to verbalize, children need a shared language. The special language of mathematics, in both symbolic and word form, should be specifically attended to (Ginsburg, 1983, 1989).

Premise 6: Embedded Assessment. The performance indicators and assessment expectations can be used for formal assessments, but they are also designed to be embedded in the informal assessments of everyday activities, in the dialogues, the questions, and the cues that teachers toss to students to help them construct new knowledge or correct pre-existing concepts (Chatterji, 2003; Solomon, 2005).

Premise 7: Representative Materials. Pictures and concrete materials, including real and representative manipulatives, increase the possibilities of mathematical perceptions. They provide useful, often indispensable, problem-solving strategies as they lead to concept formation. Manipulatives respond to individual differences in learning styles and forms of intelligence, increasing the feelings of self-efficacy for those who are more kinesthetic or tactile in their learning approach. They are particularly helpful at the early levels when children are still at concrete operational stages, and sometimes even necessary for adults whose concepts need to be redeveloped. We need, however, to remember that representative manipulatives are in essence analogies for the real thing, and conscious connections have to be made. Further transitions have to be carefully constructed as children move from the concrete materials to the symbolic forms. The words of everyday language count as well. There needs to be interactive dialogue connecting the words that explain the concept, the manipulatives, and the written symbols of the language of mathematics (Fuson & Briars, 1990).

Learners vary in their need for manipulatives and sometimes reject them once the concept or efficient procedure has been developed. They can become cumbersome when dealing with large numbers, and teachers need to use their best judgment about whether they are of value once the embedded concept is developed. A good rule of thumb is to use the materials to introduce concepts and abandon them for most students when they have made a conceptual shift to the symbolic form or operation. For some students, teachers will have to return to the concrete materials in remedial or small group sessions.

Premise 8: Problem-Solving Strategies. In addition to the use of representative materials, there are other problem-solving strategies that teachers may help students develop. In general, connections to real-life situations work as they provide motivation and help students retrieve their prior knowledge (Riley, Greeno, & Heller, 1983). Other strategies include:

- Acting out the problem with physical movements (e.g., touching each item for one-to-one correspondence)
- Making a picture, concept map, or Venn diagram
- Individual and shared analysis of problems to identify given information and desired objective
- Organizing given data on a table
- Generation, comparison, and evaluation of validity of different solution methods

- Purposeful analysis of problems based on their underlying specific concepts may also be considered a general strategy, but one that is dependent on the concepts themselves. I will make specific suggestions for these concept-based strategies in Chapter 3, but they are also embedded in the content standards.

Premise 9: Technology. Calculators and computers are not substitutes for the conceptual and procedural knowledge needed for automaticity in retrieval of basic facts, number and spatial sense, and process skills. Nor should they take away from the teacher-managed and peer-interactive discourse of doing mathematics. But they can help students build knowledge. They are, in a way, our modern algorithms—short-cut procedures for complex computations—just as the wristwatch is a technological substitute for telling the time by looking at the position of the sun. They should be considered as necessary and effective tools in learning and living, used like books, worksheets, manipulatives, balances, compasses, protractors are now, and like slide rules were in the past.

For some students, the motivation and immediate feedback of computer managed drill and practice activities will be helpful if used in conjunction with other activities. Graphing calculators that allow for quick connections between equations and graphs, and graphic drawing programs that provide easy depiction, manipulations, and transformations of figures are particularly useful. Real databanks retrieved from the Internet offer a fine supplement to the data collected by students themselves. Because of the almost universal existence of technology, it is no longer necessary for students to spend time building speed in completing multi-digit addition, subtraction, multiplication, and division algorithms. As soon as the student provides evidence of automaticity in fact retrieval, an understanding of the algorithm strategy, and reasonable accuracy, multi-digit problems should be estimated first and then done with a calculator. Some recursive practice with the algorithms can be done from time to time, but we cannot overlook the fact that being able to use technological tools is an important content standard in itself—necessary for survival in the third millennium (Solomon, 2003; University of the State of New York, 1989; Usikin, 1998).

HOW TEACHERS CAN USE ■ THE FOLLOWING CHAPTERS

A Guide for Writing Grade-Level Curriculum

As previously explained, the process standards are not separately presented in Chapters 2 and 3, but continuously embodied in the performance indicators and in the challenges of the exemplars in Chapter 3. The two listed disposition or attitude standards are similarly implied in the real-life applications of Chapter 3 and fostered by the careful attention to conceptual development in

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Chapter 2. The six major branches of the content set are presented in an order that generally corresponds to the traditional level of concentration on that branch as students progress through the grades. As a consequence, the sequence of the branches also reflects an increasing degree of mathematical complexity. For example, counting is presented first and the multiple representations of data at the end. However, each major branch has preparatory concepts at every grade level so teachers will find concepts that are appropriate for early grades in the final two branches. Within the major overlapping branches, each specific minor branch is presented in the order of a presumed developmental sequence.

When preparing grade-level curriculum, teachers should go through Chapter 2 and check off all standards appropriate for their own grade level, using the median expectations listed. There may be additional standards that are required by state documents or assessments that need to be considered, and some adjustments required by the particular group of students. The order in which the major branches are presented is optional. One branch can be presented at a time, or the teacher may choose to alternate between them. For example, the standards on the rotary clock might be a good introduction to fractions. Another alternative is to follow the order of a textbook, using all sections of this book as a side-by-side accompaniment and day-to-day reference as described below.

A Day-to-Day Reference Guide for Instruction and Informal Assessment

Once the curriculum sequence has been decided upon, teachers may use Chapter 2 as a daily reminder of what students need to know (for an in-depth discussion on assessment, see Solomon, 2002). Having the desired concept clearly in mind will help teachers construct planned and unplanned dialogues and activities that meet the needs of each student. The concept-matching and correspondingly numbered suggestions for scaffolds or instructional-mapping dialogue in Chapter 3 will help guide them in this process, but individual student's prior knowledge, motivating goals, and the teacher's own experience with successful activities should be considered. The matching exemplars can be used as they are presented and can also serve as models for the selection or creation of other similar experiences that will help the student develop a concept or automatize a procedure. The topical index for Chapters 2 and 3 will provide easy access to these for a particular lesson or unit.

The performance indicators and assessment expectations will clearly delineate the forms and measures of the informal assessments that need to be an integral part of every day's activity because they provide the feedback necessary for reflective practice. Based on the responses to these informal assessments, teachers can make day-to-day and moment-to-moment adjustments in their instructional decisions.

Formal Assessments

Formal assessments for the purpose of program evaluation at critical grade benchmarks or at the end of a particular unit of study can be constructed

directly from the performance indicators and assessment expectations in Chapter 2 and from the exemplars in Chapter 3. Formal assessments can be prepared for analysis and individualized for students as outlined below (examples of the analytical tools and reports may be found in the Resource section following Chapter 3).

- Each item on the assessment (written test or other alternative form) should be articulated with a particular standard by number.

- If possible, items should be prepared in multiple forms that reflect the mastery levels. For example, students might be able to solve a problem conceptually with concrete materials or diagrams, but be unable to translate the problem to algorithm form and solve it without materials. Clearly stated rubrics are needed for open-ended questions (see examples in the Resource section following Chapter 3).

- The expected level for each standard needs to be established. Is the expectation at the exploration level or should the concept mastery or procedural mastery level be reached? Where is each student in reference to this expectation?

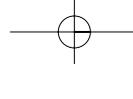
- An optional, above standard mastery level, which is not listed in Chapter 2, could be added. This might assume, for example, that the student has reached a level of the particular concept where, in addition to solving problems presented by others, the student could create new problems that require that concept or apply the concept to other contexts or interdisciplinary connections.

- Comprehensive written assessment instruments should be constructed with a balance of short and extended response, mental math, and multiple-choice items. The instrument should also consider the sequence and number of items in terms of their cognitive demand or difficulty.

- When reporting to the students themselves and their parents, the standards should be shared. A report would list the number of the standard and a rubric that corresponds to the three or four developmental levels. The achievement level reached by the student for each standard would be noted, and there would be an indication whether or not that level equaled or exceeded expectation.

- Individual student analyses and standard-by-standard analyses of class means can then provide knowledgeable direction for instruction. There are several computer-based management programs that can facilitate each of these analyses and reports. Examples of assessment items matched to standards will be found in Chapter 3. An example of a report to students or parents and a computer-based class analysis will be found in the Resource section following Chapter 3.

- An ultimate technology-based strategy that responds to assessment data would then provide teachers with interventions or instructional strategies designed to meet specifically diagnosed needs. For example, the technology would match a specifically diagnosed unfulfilled expectation or missing concept from Chapter 2 to a scaffolding dialogue or problem experience from Chapter 3. A data-based matching intervention (DBMI) system would place into the age



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of technology the best teaching strategies of individualized instruction—and perhaps finally make such instruction truly feasible in a classroom environment.

■ **NOTE**

1. The standards of the National Council of Teachers of Mathematics (2000) are the basis for the standards presented in this text, but the presentation differs in that it relates the standards to the forms of knowledge and its acquisition, presents the processes first, adds perception and attitude standards, and includes succinct definitions of embedded terms. For greater elaboration and examples see the standards themselves.

