
Mastery Strategies

OVERVIEW

Mastery strategies help students *remember* mathematical content and *procedures* and *practice* their *computational skills*. They are especially engaging to Mastery math students.

Mastery math students . . .

- *Want to* learn practical information and set procedures.
- *Like math problems that* are like problems they have solved before and that use algorithms to produce a single solution.
- *Approach problem solving* in a step-by-step manner.
- *Experience difficulty when* mathematics becomes too abstract or when faced with nonroutine problems.
- *Want a math teacher who* models new skills, allows time for practice, and builds in feedback and coaching sessions.

The six Mastery strategies in this chapter can help you meet these NCTM Process Standards (see Figure 1.0).

Strategy	NCTM Process Standards*				
	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Convergence Mastery [p. 15] – Students prepare individually and take quizzes with peer review until all students demonstrate complete mastery of the content.			✓		
Vocabulary Knowledge Rating [p. 18] – At different points throughout a unit, students rate their knowledge of critical vocabulary terms.			✓		
Proceduralizing [p. 24] – Students internalize a procedure by observing their teacher demonstrating it, writing its steps in their own words, and using it to solve problems cooperatively and individually.	✓		✓		
Mental Math Strings [p. 30] – For a few minutes each day in class, students are challenged to perform mathematical operations and solve problems mentally.		✓		✓	
Graduated Difficulty [p. 36] – Students assess their level of competence by successfully completing a task from an array of options at different levels of difficulty.	✓	✓			
New American Lecture [p. 42] – Students are “hooked” into a presentation and use a visual organizer and deep-processing questions to make notes, organize information, and remember essential content.			✓	✓	

FIGURE I.0 Correlation of Mastery Strategies to NCTM Process Standards

*For more information on the National Council of Teachers of Mathematics (NCTM) Process Standards, please consult their *Principles and Standards for School Mathematics* (2000), or visit their website at www.nctm.org.

Convergence Mastery

Strategy Overview

In every math classroom, after every quiz or test, a wonderful instructional opportunity presents itself: the opportunity to help students learn from their mistakes. But the truth is, in most math classrooms, tests are returned with red marks on them, and the next unit begins. This means that the great majority of math students are missing the chance to root out errors, clarify confusions, and grow as learners.

The Convergence Mastery strategy is a simple but powerful way to provide students with multiple opportunities to learn from their mistakes and achieve mastery of important math procedures and skills. At the heart of the strategy is a series of short quizzes focused on a single core skill (e.g., factoring polynomials). Before taking the first quiz, students practice the skill and review in pairs or small groups. Students take the first quiz individually, return to their groups, and grade one another's quizzes as the teacher provides the correct answers. Only two grades are possible:

1. Students with one or more incorrect answers receive an *Incomplete*
2. Students who answer every question correctly receive an *A*

Students who receive an *A* are not required to take additional quizzes. Instead, they help group members who received an *incomplete* to review, make corrections, and prepare for the next quiz. The strategy continues until all students have received an *A*.

How to Use the Strategy

1. Select a math procedure or skill you want all students to master.
2. Develop three to five short quizzes that contain problems representative of the skill.
3. Explain the quiz process and grading procedures to students. Be sure they understand how the process is designed to help them.
4. Provide a few minutes for students to review the skill in small groups.
5. Administer a quiz to all students. Quizzes are timed (usually 5 minutes per quiz).
6. Share the correct answers, and have students grade other group members' quizzes.
7. Excuse any students who received an *A* from further quizzes. Have these students help their group members correct errors and prepare for the next quiz. If necessary, provide coaching sessions to struggling students.
8. Continue the process (Steps 5–7) until virtually all students have received an *A*.

The Strategy in Action: Examples

Figure 1.1 shows sample topics ideal for the Convergence Mastery strategy.

Numbers	Algebra	Geometry	Measurement	Probability	Statistics
Operations on whole numbers	Evaluating expressions	Analysis of plane figures	Calculating perimeter	Finding P(A)	Computing mean, median, and mode
Operations on fractions	Solving equations	Computing numbers of diagonals	Calculating area	Finding P(A or B)	Interpreting histograms
Operations on integers	Factoring expressions	Transforming figures	Calculating surface area	Finding P(A and B)	Creating box-and-whisker plots
Prime factorization	Working with irrational numbers	Drawing nets of 3D figures	Converting metric units	Permutations and combinations	Interpreting graphs

FIGURE 1.1 Ideal Convergence Mastery Topics

Figure 1.2 shows a sample set of quizzes for solving equations.

Quiz 1	Quiz 2	Quiz 3	Quiz 4	Quiz 5
Solve each equation and show your work.				
1. $2(4x + 5) = 26$	1. $2(6x + 4) = 32$	1. $5(2x + 5) = 35$	1. $4(2x + 3) = 36$	1. $6(x + 2) = 48$
2. $.5(6x - 8) = 15$	2. $.5(8x - 3) = 7$	2. $.5(4x - 8) = 10$	2. $.5(3x - 1) = 11$	2. $.5(10x - 8) = 11$
3. $-3(3x + 2) = -24$	3. $-4(x + 5) = -60$	3. $-2(3x + 7) = -4$	3. $-3(2x + 1) = -27$	3. $-2(4x + 8) = -10$

FIGURE 1.2 Sample Convergence Mastery Quizzes—Solving Equations

Why the Strategy Works

Sometimes, teaching strategies come from real-life experiences. The initial seed for the Convergence Mastery strategy was planted at Boy Scouts camp where one of the authors of this book (Ed Thomas) spent his summers as a young boy. At summer camp, scouts had opportunities to earn awards and merit badges for meeting various challenges. One challenge was to make fire without matches. On certain nights, scouts were given a rock, a piece of steel, and 30 minutes to produce a fire. Scouts who succeeded were given a “Singed Eyebrows” certificate. Scouts who failed were invited to try again the next time the Singed Eyebrows station was open. Whenever a scout produced fire, whether it was on the first try or the fourth, he received his Singed Eyebrows certificate. This open-door policy on success motivated scouts to keep trying, learn from their mistakes, and achieve mastery in the skill of making fire.

Convergence Mastery takes the wisdom of Boy Scouts camp and puts it to work in the mathematics classroom. The strategy provides students with repeated and controlled practice opportunities, which help build students' debugging skills and maximize skill acquisition. Convergence Mastery also provides teachers with an easy way to differentiate instruction according to students' readiness levels. Students who need more practice opportunities and more coaching receive both. At the same time, students who have already mastered the skill do not sit around idly; instead, they become part of the teaching and learning process. What's more, by having students help other students who have yet to receive an A on a quiz, Convergence Mastery capitalizes on the power of peer-coaching partnerships, which have been shown to increase students' academic intensity (Fuchs, Fuchs, Mathes, & Simmons, 1997) and lead to academic gains and more positive attitudes toward subject matter (King-Sears & Bradley, 1995).

Planning Considerations

The idea behind Convergence Mastery is that all students *converge towards mastery* of the highlighted skill or procedure by achieving a perfect score on a short quiz. In terms of preparing for Convergence Mastery in the classroom, most of the planning time goes to the development of the quizzes. Here are a few guidelines to keep in mind when developing Convergence Mastery quizzes:

Select a focus skill that students are familiar with and have partially mastered. For example, if students have experience with solving linear equations but have been making mistakes in the process, then solving linear equations would be an ideal candidate for Convergence Mastery.

Make the quizzes brief. Remember, during a single class period, students will be taking up to five quizzes while also spending 5 minutes of study time between each quiz. Make sure that each quiz can be completed by students in 5 minutes.

Keep the focus skill and the level of difficulty constant across all the quizzes. Only the problems should vary from quiz to quiz.

Consider projecting the quizzes. To save paper and time, you might consider writing the quizzes on a transparency or designing them in a program like Microsoft PowerPoint and then projecting them through a multimedia device.

Variations and Extensions

Depending on the difficulty of the focus skill and students' level of proficiency, you may choose to play a more active teaching and coaching role between quizzes. If you choose to run the between-quiz coaching sessions yourself, you may want to vary the role of students who earn the A grade and exit the quiz-taking activity. For example, you might allow them to begin their homework, or ask them to design and solve problems at a higher level of difficulty than those on the quizzes. These students can also be invited to participate in subsequent quizzes and earn extra credit points, which increases the sense of reward for having mastered the skill early.

Vocabulary Knowledge Rating (VKR)

Strategy Overview

Successful students know how to assess and evaluate their own learning. They tend to have a clear understanding of what they know and which concepts and ideas they still need to learn. Vocabulary Knowledge Rating (VKR) gives teachers of mathematics a strategic approach to vocabulary instruction—an approach that helps students evaluate the state of their learning and build a deep understanding of critical content.

VKR provides teachers with a wealth of formative assessment data by providing answers to two questions: One, which concepts are giving the entire class difficulty? And two, which individual students are struggling most with the content of the unit?

In VKR, students numerically rate their understanding of key terms using an organizer like the one shown in Figure 1.3.

Fractions				
Vocabulary Term	I've never heard of the term	I've seen or heard of the term before	I think I know the term	I know the term and can explain it
common denominator	1	2	3	4
denominator	1	2	3	4
equivalent fractions	1	2	3	4
fraction	1	2	3	4
improper fraction	1	2	3	4
inverting	1	2	3	4
least common denominator	1	2	3	4
mixed numbers	1	2	3	4
numerator	1	2	3	4
proper fraction	1	2	3	4
My Vocabulary Knowledge Rating:		25	Today's Date: Oct 8, 2009	

FIGURE 1.3 Student's VKR for a Unit on Fractions

Because self-assessment is always an ongoing process, VKR is most effective when it is used regularly. Typically, students complete a VKR organizer at least three times over the course of a mathematics unit:

- *Before the unit begins* to assess their initial understanding and help them activate any relevant background knowledge.
- *During the unit* to assess what they currently know and understand and to determine which words and concepts require more study.
- *After the unit is completed*, but prior to the test or culminating assessment, to focus study efforts and to reflect on the learning process.

How to Use the Strategy

1. Prioritize your vocabulary by selecting the 10 to 12 most important words from your unit that students should focus on. Limit your selections to only the most critical words that every student will need to know and understand.

2. Distribute a VKR organizer to each student (see Organizer A on page 23 for a blank reproducible). Review the ranking system with students:

1 = I have never heard of this term.

2 = I have seen or heard of this term, but I am not sure what it means.

3 = I think I know what this term means.

4 = I know this term, and can explain what it means.

3. Have students rate their current knowledge of each vocabulary word by selecting the appropriate number on the four-point scale. To complete the organizer, students record the sum of their points in the Knowledge Rating box and date their work.

4. Have students revisit their initial VKR organizers throughout the unit to reassess their knowledge and monitor how their understanding of key content has expanded and still needs to grow.

5. Help students prepare for an end-of-unit test or culminating assessment by giving them time to review and reflect on their previously completed VKR organizers and to discuss their learning with classmates.

Implementation Note: Formative assessment data from students' VKR organizers can also be used to provide parents and educational support service specialists with valuable information to help focus conversations with each student about his or her learning.

The Strategy in Action: Examples

Here are some sample middle school mathematics units with corresponding vocabulary terms:

Measurement: area, altitude, base, circumference, diameter, grid, length, unit of measurement, perimeter, radius, volume

Geometry: circle, hexagon, octagon, parallelogram, pentagon, polygon, quadrilateral, rectangle, rhombus, square, trapezoid, triangle

Statistics: box and whisker plot, data, mean, measures of central tendency, median, mode, range, quartile, rank, stem and leaf plot, tally, variance

Why the Strategy Works

In their research into vocabulary instruction, Jenkins, Stein, and Wysocki (1984) show that students need to be exposed to new words at least six times to master and retain their meanings. VKR provides teachers with a manageable way to keep students closely connected to the key terms and concepts in a unit, giving them the exposure they need to learn new words deeply.

Another important aspect of vocabulary instruction is focusing on only the most important concepts and terms. In fact, Marzano (2004) shows that when vocabulary instruction is focused on critical academic terms (as opposed to high-frequency word lists), student achievement can increase by as much as 33 percentile points on content-area tests. This is why VKR concentrates both the teacher's and students' attention on only 10 to 12 terms.

A final benefit of VKR is that it builds the habits and skills of self-regulated learning, which has been identified as a hallmark of intelligent behavior (Costa & Kallick, 2000).

Planning Considerations

While VKR is not a difficult strategy to implement, there are a few important guidelines to consider:

Select only the most critical words. Resist the temptation to list every word that students might encounter during the unit, and include only the 10 to 12 critical words that will be most helpful to students. In some cases, you may choose to include words that are not quite central to the unit but will help scaffold student learning. For example, in selecting words for a unit on fractions, a teacher chose to include the term *number line* because she wanted students to compare and order fractions using a number line.

Determine when and how you will help students assess their learning. Identify the segments of your unit in which you will present and discuss a number of the words and concepts from your list. In between these segments, give students ample opportunity to self-assess their knowledge

and reflect on their progress. Make sure that all students know that these are valuable periods of class time that should be used for meaningful reflection and discussion.

Decide when it will be appropriate to analyze students' progress. Ask yourself, At what points would it be helpful for me to know whether students have learned the key words and concepts? After a weekend? Before a school break? At the completion of smaller sections within the unit?

Variations and Extensions

Create Your Own VKR

After you and your students have used a traditional VKR organizer, consider inviting your class to revise or improve upon the form by creating a new set of descriptive column headings. For example, after a group discussion, a class of sixth graders developed a three-point VKR scale with the following headings:

- 1 = I really don't know this word.
- 2 = I have seen or heard this word.
- 3 = I really know this word because I can give an example.

You can also extend the strategy by inviting students to develop their own creative, personally meaningful VKR organizers. For example, Figure 1.4 shows how an eighth grader incorporated her love of softball into her VKR organizer.

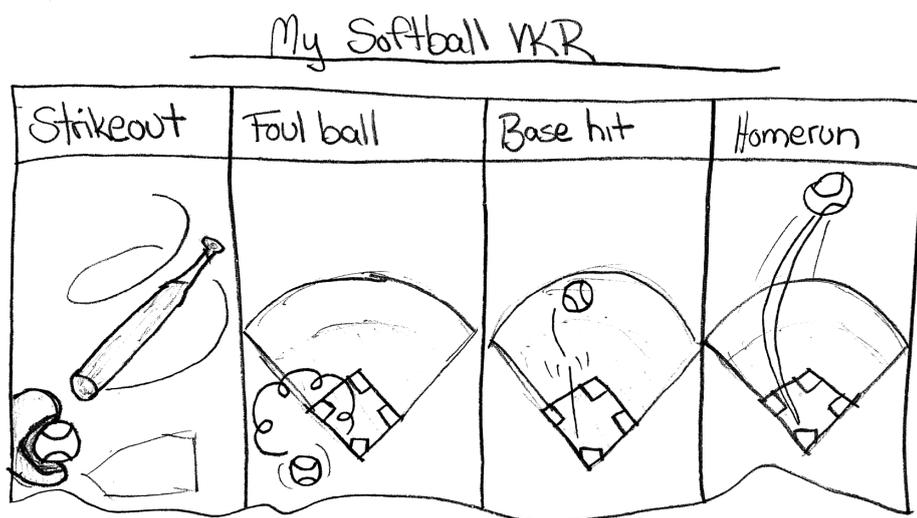


FIGURE 1.4 Student's VKR Using Softball Icons

Definition Doctor

Definition Doctor (adapted from Thomas, 2008) is a great way to review VKR words and to formatively assess students' vocabulary knowledge using an engaging whole-class game format. It works especially well in conjunction with VKR because VKR automatically aligns the activity with a set number of critical terms that have already been the focus of instruction. You can use Definition Doctor at various points within the VKR cycle by following these simple steps:

1. Have all students take out their VKR organizers.
2. Begin by playing the role of the Definition Doctor yourself. Select a student (Student A) to choose one word for you—the Definition Doctor—to define and explain why the word is important to the lesson or unit.
3. After you define and explain the word, Student A assumes the role of the Definition Doctor and a new student (Student B) chooses a word for the good doctor to define and explain. (If the Definition Doctor is having trouble, you may choose to allow him or her to get a “second opinion” by allowing for a “consult” with you or another student.)
4. Student B now becomes the Definition Doctor and a new student selects a word.
5. Continue this process until all the words from the list have been defined and explained.
6. Keep track of any words that seemed to give students difficulty and review them with the class.

Organizer A: Vocabulary Knowledge Rating Organizer

Topic:				
Vocabulary Term	I've never heard of the term	I've seen or heard of this term before	I think I know this term	I know the term and can explain it
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
	1	2	3	4
My Vocabulary Knowledge Rating:			Today's Date:	

Proceduralizing

Strategy Overview

Mathematics is brimming with procedures: long division, finding the greatest common factor, completing the square, differentiating the product of two functions, just to name a few. Often, the difference between successfully and unsuccessfully internalizing mathematical procedures proves the difference between high and low achievers in mathematics. In classrooms throughout the country, teachers of mathematics spend significant classroom time demonstrating procedures while students watch, copy their teacher's work, and strive to keep pace before trying a few practice problems on their own.

But jump ahead a few weeks and many students won't be able to recall the steps well enough to apply the procedure (or skill) and successfully solve a problem. In order for students to master, internalize, and retain the steps of important mathematical procedures, students need to do more than watch, copy, and keep pace with their teacher's work; students need to make mathematical procedures their own.

The Proceduralizing strategy helps students make even the most challenging mathematical procedures their own through observation, mathematical analysis, collaboration, and independent practice. Students observe a teacher solving different sample problems using the procedure; analyze the procedure by identifying its general steps and writing the steps in their own words; collaborate with a partner to solve two problems, once by coaching and once by calculating; and independently practice and refine their knowledge by applying the procedure to solve a set of problems.

How to Use the Strategy

1. Select a mathematical procedure that is important for your students to learn. The procedure should be accessible to students and be relevant to current (or future) classroom applications.
2. Model the procedure with students using sample problems. The sample problems should not be overly complex; rather, they should clearly illustrate the general steps in the procedure.
3. Review the sample problems with students, and focus on the essential steps in the procedure. Don't overwhelm students with additional information like prerequisite procedures or secondary processes.
4. Work with students to identify a set of generalized steps for the procedure. Have students write these steps in their own words. Encourage students to ask questions about the procedure and steps to help them internalize the information.
5. Organize students into pairs. Have students review the procedure together to make sure that they've internalized all of the steps.

6. Provide each pair of students with two problems. One student works on solving the first problem (without seeing the steps) while the other student coaches using the steps. For the second problem, students switch roles. While students are working collaboratively, circulate around the room to monitor students' progress and answer any questions.
7. Encourage students to share their experiences with the entire class, including both successes and difficulties they had solving the problems collaboratively.
8. Assign additional problems for students to solve independently using the same procedure. This work can be completed in class or for homework.

The Strategy in Action

Jackie Parra teaches sixth-grade mathematics and has identified graphing equations of the form $y = mx + b$ as an important procedure for all of her students to know well. To help her students master this essential graphing procedure, Jackie uses the Proceduralizing strategy.

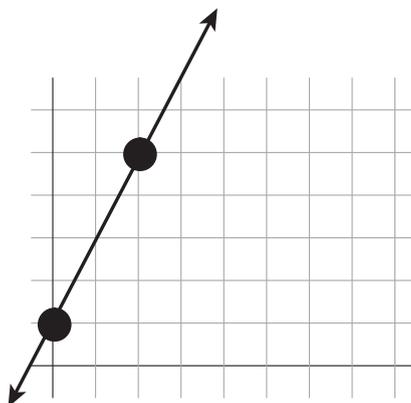
Jackie starts by graphing a few linear equations using the slope-intercept form, $y = mx + b$, and summarizes the procedure into five general steps that her students can easily understand:

1. Write the equation in the form $y = mx + b$.
2. Use graphing paper to plot the number b on the y -axis. (This point is the y -intercept of the graph.)
3. Write the slope m as a fraction $\frac{k}{h}$.
4. From the y -intercept, move h units horizontally and k units vertically on the plane. Plot a point at this location.
5. Draw a line (using a straight edge) through the two points plotted in Steps 2 and 4. The line, and all of its points, represent the infinite set of ordered pair solutions (x,y) for the linear equation $y = mx + b$.

To better illustrate the steps in the procedure, Jackie models one more equation for students. While she graphs this equation step by step, her students carefully record the steps in their own words. Jackie reminds her students to write the steps in general terms so they can use them to graph other equations (and coach their partners).

1. Beginning with the equation $2x - y = -1$, Jackie rewrites the equation in $y = mx + b$ form: $y = 2x + 1$.
2. Second, Jackie plots the y -intercept (b) on a transparency that she projects onto the board: $(0,1)$.
3. Third, she reminds students to be careful and make sure that they write the slope (m) as a fraction $(\frac{k}{h})$: $m = 2 = \frac{4}{2}$.

4. Fourth, Jackie returns to her point on the plane. She moves four units up (k) and two units to the right (h). She plots a second point here: $(2,5)$.
5. For the fifth step in the procedure, Jackie uses a straightedge to draw a line between the two points: *line passing through $(0,1)$ and $(2,5)$.*



She explains to her students that for every equation of the form $y = mx + b$ there is an infinite number of ordered pairs of numbers that can be substituted for x and y to form a true equation. The set of all of the ordered pairs of numbers that make the equation true is the solution of the equation. When graphed, these points representing the solution of the equation will form a straight line.

After she finishes graphing the equation $y = 2x + 1$, Jackie organizes her students into pairs. Students share their steps with their partners and review the procedure before starting the activity together.

Jackie provides each pair of students with two equations to graph. One student puts away her steps and graphs the first equation while her partner coaches her. Students switch roles for the second equation. The student-coach from the first equation now graphs the second equation while his partner coaches him.

After all of the students have had the opportunity to graph an equation and coach their partner doing the same, Jackie brings her class back together. She asks questions and encourages her students to share their thoughts on the Proceduralizing strategy so far. Jackie wraps up her lesson by assigning students some more equations to graph for homework so they can practice using the procedure on their own.

Why the Strategy Works

Proceduralizing draws its instructional power from two distinct lines of research: direct instruction and peer coaching. Let's begin with direct instruction. Direct instruction is a broad name for those teaching frameworks that involve the teacher modeling a skill or procedure for students. After the teacher has modeled the skill, students practice it in phases, with

less help and guidance from the teacher during each phase. The ultimate goal of a direct-instruction lesson is student independence.

Research has consistently shown that direct instruction has a dramatic impact on students' mathematical achievement, improving students' ability to master and retain procedures, solve problems with greater confidence, and remain focused and engaged in mathematical learning (Kroesbergen & Johannes, 2003; Flores & Kaylor, 2007).

However, we have all probably used direct instruction in our mathematics classrooms and been less than thrilled by the results. The reason for this is simple: Not all forms of direct instruction are created equal. Proceduralizing maximizes the benefits of direct instruction by adding several additions and twists to the traditional direct-instruction model. Figure 1.5 outlines these additions and twists and compares Proceduralizing to more traditional direct-instruction methods.

Traditional Methods	Proceduralizing Strategy
<ul style="list-style-type: none"> • Teacher models a procedure or skill. • Students copy the steps in the procedure verbatim. • Students are given practice problems to work on. • Students can ask questions about the process. • Students complete additional problems for homework. 	<ul style="list-style-type: none"> • Teacher models a procedure or skill to the class using sample problems. • Teacher reviews the sample problems with students. • Students analyze sample problems and begin to personalize the general steps in the procedure. • Students record the steps in the procedure in their own words. • Students work collaboratively in pairs to solve two problems, once by following the steps in the procedure and once by coaching their partner. • Students are encouraged to ask questions and share their experiences during class discussion. • Students complete additional problems independently, either in class or for homework.

FIGURE 1.5 Traditional Direct Instruction Versus Proceduralizing

Of all the revisions to traditional direct instruction, the most important is the integration of a simple peer-coaching model into the larger direct-instruction framework. By allowing students to work and learn together in a structured way, you and your students reap the significant benefits associated with learning partnerships, including more on-task behavior, increased engagement, and the development of more positive attitudes towards mathematics (King-Sears & Bradley, 1995).

Planning Considerations

Mathematics contains a great number of procedures. Some procedures are more complex with numerous steps while others are more basic and contain only a few steps. Some procedures are essential to mathematics while others aren't as important and are "nice to know."

While the Proceduralizing strategy works best with critical procedures involving multiple steps, it can be used with most levels of mathematical content. When planning a Proceduralizing lesson for your classroom, you should follow these guidelines:

- **Select a procedure that maintains a high degree of consistency for a variety of problems for which the procedure applies.** For example, the procedure for finding the least common multiple (LCM) or greatest common factor (GCF) of a pair of numbers remains consistent for every pair of numbers.
- **Review prerequisite skills with students, but don't present too much information.** Most mathematical procedures rely on concepts and skills that students have learned previously. However, it is important that some time be taken to identify and review relevant concepts and skills so they do not become barriers for students. It is also important not to present so much information that the key steps in the procedure are lost.
- **Work through the procedure and generalize the steps before modeling the procedure with students.** Students are expected to generalize and record the steps of the procedure in their own words, so it is important that you have a clear set of general steps. By working through the procedure and developing a list of steps in the planning stage, you will be prepared to review the procedure, answer students' questions, and provide help and feedback as needed.
- **Make sure your examples are clear, concise, and easy to understand.** Modeling the procedure is essential to the Proceduralizing strategy. You need to consider how information will be presented to and received by students. Any written information needs to be visible and readable for all students. Manipulatives and hands-on procedures need to be easily seen and connect well to your mathematics content. Your modeling of the procedure should be clear, efficient, and, of course, mathematically accurate.
- **Allow ample time for students to write their steps in their own words, share their steps, and review the procedure with a partner.** Cognitively speaking, the most important part of the Proceduralizing activity is the period of time in which students write the steps in their own words and refine their steps with a partner.
- **Prepare appropriate problems for the cooperative learning activity.** Be sure that the problems you select are aligned with the procedure that your students are learning. Serving up problems that don't fit the procedure will deflate the entire learning experience.

Similarly, problems that are only somewhat related to the procedure can be confusing at first and should be introduced later on in the unit.

- **Select an appropriate independent practice activity to close the lesson.** Think about how you want your students to practice the procedure on their own. Assign activities for class work or homework that are meaningful and will help students further internalize the steps in the procedure.

Variations and Extensions

While the Proceduralizing strategy works best with core mathematical procedures involving multiple steps, the following moves can help you use the Proceduralizing strategy with any mathematical content.

Fill in the Blank

For younger students or students who are struggling with a procedure, you can provide students with your typed list of general steps for the procedure with a few key words missing. As students watch and study your demonstration of the procedure, they fill in the blanks of each step and capture the key elements of the procedure.

Math Vocabulary: Fun and Games

Sometimes a new mathematical procedure includes a significant number of new vocabulary words. Make sure you preview these words and define them with your students prior to starting your Proceduralizing lesson. Vocabulary games are great ways to introduce students to new and potentially intimidating terms.

Three's Company

The Proceduralizing strategy can easily work with cooperative groups of three students. After students review their steps in the procedure, they are given three problems. While one student works to solve the first problem, the other two students observe and coach using their steps. Students switch roles, so each student solves one problem and coaches twice.

Apply Technology

To verify that the procedure really does work, have students use a computer or calculator to check for themselves. For the graphs of $y = mx + b$ equations featured earlier, students could use a graphing calculator to check that their solutions and graphs are indeed correct. Also, consider using software programs to introduce your students to a procedure. A slide show is a great way to show visual representations of the procedure and reveal the steps in the procedure one at a time.

Mental Math Strings

Strategy Overview

Most math educators would agree that *in order to learn math, students must do math*. This principle, put forth by the National Council of Teachers of Mathematics (NCTM) in 1989, has served as a cornerstone for reforming mathematics instruction ever since. Today, many math educators use the term *engagement* when they talk about students doing math in the classroom.

Mental Math Strings is a daily activity that engages students directly by challenging them to call up essential facts and perform a wide variety of mathematical operations—to do math—in their heads. No calculators, no paper, no pencils. The strategy requires only a few minutes each day and can be done at the start of class, end of class, or as a break during class. We like to think of Mental Math Strings as a kind of a daily multiple vitamin for math students: They provide a rich mix of nourishing mathematical concepts and activities that help keep students' minds in peak condition.

A typical Mental Math String includes key vocabulary terms, important measurement equivalencies, and connections to important mathematical concepts. For example, a Mental Math String might contain this sequence of activities:

- Start with the number of inches in two feet.
- Add the first odd whole number to your answer.
- Divide by the number of vertices in a pentagon.
- Cube your result.
- Add the digits of your answer together.
- Multiply by the number of diagonals in a square.

So, how did you do with this Mental Math String? Did you come up with an answer of 16?

While a simple Mental Math String is beneficial to students, the daily string routine can be greatly enhanced using an instructional model called PEACE.

The progressive stages of the PEACE model ask students to

Preview the material;

Engage in the mental operations required by the string;

Assess their final answers;

Correct mistakes and errors in thinking; and

Engage in a second Mental Math String.

How to Use the Strategy

To implement Mental Math Strings in your classroom, follow the stages in the PEACE model:

Preview Stage: Preview the content, procedures, and skills embedded in the Mental Math String with your students.

Engagement Stage 1: Present the steps of the Mental Math String one at a time to students so they can compute each step in their heads.

Assessment Stage: Provide the final answer, so students can determine whether they performed all of the steps correctly.

Correction Stage: Demonstrate (verbally, in written form, or using a student volunteer) how each step is correctly performed, allowing students to identify and correct mistakes one line at a time.

Engagement Stage 2: Present a second Mental Math String that involves the same concepts, vocabulary, and procedures but uses different numbers. Collect work from all students to note and communicate rates of improvement between the first and second string.

The Strategy in Action

Joanne Ehrhardt uses Mental Math Strings to engage students in mathematical learning and continuously build their background knowledge of key vocabulary terms, measurement units, and mathematical procedures.

In the preview stage, Joanne previews the information that the students will encounter in the steps of the Mental Math String. Prior to today's Mental Math Strings challenge, Joanne asks students to activate and shore up gaps in their prior knowledge with these questions:

1. How many inches are in one foot?
2. How many inches are in one yard?
3. What is the first odd whole number?
4. What is the first even whole number?
5. How many sides are on a pentagon? On an octagon? On a hexagon?
6. How many vertices are on a pentagon? On an octagon? On a hexagon?
7. How do you square a number?
8. How do you cube a number?
9. What is the sum of the digits of the number 64? What is the product?
10. How many diagonals can be drawn on a square? On a parallelogram?
On a rhombus?

The preview stage of the PEACE model provides students with opportunities to learn and review important mathematical facts and operations

that will be part of the Mental Math String, along with related facts that will not explicitly appear in the string. Previewing in this way also increases students' chances of succeeding in the Mental Math Strings activity and builds students' knowledge base of critical facts and procedures.

After the preview stage, Joanne and her students enter the first engagement stage. Joanne reads each line of the Mental Math String and pauses for a few seconds after each line, giving students time to recall or determine the key number associated with that line of the string and then apply that number as an operation of the previous answer. For example, here is today's Mental Math String (Figure 1.6):

Today's Mental Math String

Start with the number of inches in three feet.	Mental Math: $3(12) = 36$
Add the first odd whole number to your answer.	Mental Math: $36 + 1 = 37$
Subtract the number of sides on a hexagon.	Mental Math: $37 - 6 = 31$
Add the digits of your answer together.	Mental Math: $3 + 1 = 4$
Square your answer.	Mental Math: $4(4) = 16$
Divide by the number of diagonals in a rhombus.	Mental Math: $16 \div 2 = 8$
	Final Answer: 8

FIGURE 1.6 Today's Mental Math String

In the assessment stage that follows, Joanne invites students to turn to one another and share their final answers, and then she encourages students who feel confident about their answers to share them with the class. To alleviate any confusion or differences of opinion, Joanne quickly identifies the correct answer.

Students who did not solve the Mental Math String correctly now get an opportunity to identify what they did wrong during the correction stage of the PEACE model. To accomplish this, Joanne asks for a volunteer who solved the string correctly to stand before the class and verbally recount the steps in the string so all students can identify their mistakes one line at a time. Sometimes, the volunteer student will record the line-by-line answers on the board to make it easier for students to identify their mistakes. Joanne is always pleased by how eager students are for this opportunity to receive immediate feedback on their work. She is also very happy that the number of student moans and groans over their own mistakes and miscalculations has dropped significantly since the beginning of the year. Since using Mental Math Strings, Joanne's students are more careful, more confident, and more knowledgeable than they were at the start of the year.

During the final phase of the PEACE model, the second engagement stage, Joanne encourages students to show that they have learned from their mistakes and helps all of her students experience success. Joanne presents a second Mental Math String, which includes the same vocabulary terms and concepts as the first but contains different numbers and requires different mental calculations.

Students complete the string, check their answers, and the day's lesson begins. Before moving on to the day's lesson, however, Joanne is sure to remind students that Mental Math Strings are daily exercises and that they will work on another string tomorrow to help keep their mathematical minds sharp.

Why the Strategy Works

Some of the most striking findings in recent educational research come from studies into the power of formative assessment. For example, Robert Marzano (2006), citing a 1991 study conducted by Bangert-Drowns, Kulik, and Kulik, shows that regular formative assessment can lead to gains of 25 percentile points or more in student achievement.

So, what makes for effective formative assessment? According to Marzano, the most important criterion is "sound feedback," or feedback that

- Is frequent;
- Gives students a clear idea of how well they are learning and how they can get better; and
- Provides encouragement to the student. (Marzano, 2006, p. 11)

Mental Math Strings put the power of formative assessment and sound feedback to work in the classroom. It happens daily. It provides both the teacher and the student with good information about students' understanding

of key mathematical terms and their fluency in procedures that they need to be able to perform automatically to achieve success in mathematics—all without putting undue burden on the teacher. And Mental Math Strings encourages students by giving everyone in the class the opportunity to achieve success by systematically rooting out errors in their own thinking and then putting their revised understanding to work with a second Mental Math String.

Planning Considerations

In designing Mental Math Strings, keep the following tips in mind:

1. Include terms and procedures that are central to what you're teaching. Mental Math Strings typically require general mathematical knowledge as well as more unit-specific terms and procedures. For example, the Algebra String below focuses on the terms *greatest common factor* and *coefficient*. It also requires students to apply the procedure for finding a GCF in their heads.

- Start with the greatest common factor (GCF) of 24 and 30.
- Square your result.
- Add the coefficient of $4x^5$.
- Subtract the number of degrees in a right angle.
- Square your result.

2. Keep it short and sweet. Mental Math Strings are meant to be used daily and completed quickly. Five to eight steps in a string are all you need.

3. Make two. Remember, you'll need a string for both engagement stages in the PEACE model.

4. Decide how to preview the string. Once you've identified the vocabulary, concepts, skills, and procedures students will need to know to succeed, think about what you will do to preview the activity. Will you provide questions to students as Joanne Ehrhardt did in the model lesson? Will you review and reteach the component parts? Can you involve "student-teachers" in the preview stage?

Variations and Extensions

PEACE Model

The PEACE model can be applied to any engagement activity in any mathematics classroom. Whether students are learning computation procedures with rational numbers, computing perimeters, areas, surface areas, and volumes in geometry, or solving equations in algebra, the progressive stages associated with the PEACE model will increase success rates and student learning.

The PEACE model can also be applied to homework. Students will experience more success with homework if the associated mathematical terms and procedures are previewed before students begin their homework. This might be done towards the end of class on the same day that the homework is assigned. It is equally important that the students' work is assessed the day the homework is due. Following the assessment, students should be led through the process of identifying and correcting their mistakes. Similar problems should be part of the next homework assignment so students can demonstrate that they have learned from their mistakes and can enjoy higher levels of success.

While the PEACE model described in this section includes two engagement stages, additional engagement stages can be added as needed. It is vital to the success of the strategy that the assessment and correction stages are implemented after each student engagement stage.

A final and highly engaging variation on Mental Math Strings involves student-generated strings. Challenge students to use the content they're currently learning to create Mental Math Strings for their fellow students. Students can then work in pairs or small groups to complete each other's strings, review key processes and concepts, and work through the PEACE model (or use a simplified version of the PEACE model).

Graduated Difficulty

Strategy Overview

In the mathematics classroom, students function at different levels of proficiency and comprehension. Some students may not be ready for the most challenging problems, while others become bored with problems and concepts that they have already mastered. This means that when math teachers rely on one-size-fits-all teaching and problem-solving approaches, students at both the higher and lower levels of proficiency will likely become frustrated and may disengage from the learning at hand.

The Graduated Difficulty strategy provides an effective remedy to this common classroom challenge. For a Graduated Difficulty lesson, the teacher creates three levels of problems, all representing the same math concept or skill, but at distinct levels of challenge. The first level requires students to demonstrate basic knowledge, understanding, and proficiency associated with the concept or skill. The second level includes an extension or challenge that requires students to apply their knowledge, understanding, and proficiency beyond the basic level. The third level calls for the application of higher levels of math reasoning within or even beyond the context of the math concept or skill.

Before beginning a Graduated Difficulty lesson, it is important to explain to students the value of self-assessment and to remind them that they are responsible for their choices as they

- Analyze the three tasks;
- Select the task that best fits their level of understanding;
- Complete the task and assess their performance; and
- Set goals for achievement at higher levels.

How to Use the Strategy

1. Select a math concept or skill you want your students to master.
2. Develop three problems or problem sets that represent three levels of difficulty.
3. Explain the process and the value of accomplishment associated with the varying levels of difficulty. Make sure students understand that three levels of difficulty are provided so they can analyze their own skill and comprehension levels, make choices, succeed, advance to higher levels, and get the most out their learning experience.
4. As students analyze the different problems, encourage students who are capable to select the more challenging levels, and assure all students that it's okay to begin with the easier problems and to switch levels during the activity.
5. Provide an answer key (or rubric) so students can check their work. Students who successfully complete the level three problems can

serve as coaches for other students, or they can design even more challenging problems of their own and then solve them.

6. After all the students complete their work, invite students to present their solutions to the class.
7. At the conclusion of the Graduated Difficulty activity, help students establish personal goals for improvement. Provide additional tasks or problems so students can build their knowledge, understanding, and proficiency. The additional practice can come in the form of in-class work or homework.

The Strategy in Action: Examples

Three Levels of Problem Sets Involving Integers

<i>I. Simplify.</i>	<i>II. Simplify.</i>	<i>III. Simplify.</i>
1. $12 - 15$	1. $-10 + 14 - 20$	1. $-12 - (-20)$
2. $-21 + 11$	2. $24 - 6 - 16$	2. $14 + 24 - 30$
3. $32 + 14 - 22$	3. $12 - 30 + 15$	3. $-12 - 0 + 14$
4. $-14 - 18$	4. $17 - 21 - 4$	4. $-24 - (-20) - 11$
5. $-25 + 15$	5. $-8 + 11 - 20$	5. $19 - 30 - 6 + 14$

Three Levels of Word Problems Involving Percents and Discounts

Level 1: High Tech Super Store has a 25% off sale on all computers and printers. Travis bought a computer and laser printer that regularly cost \$799 and \$249, respectively. Disregarding tax, find the total discounted cost of the new computer and printer.

Level 2: The Johnsons are starting a new family business and need to purchase two computers and two printers. Electronics and More is having a major sale. All computers are being discounted by 25% and all printers are being discounted by 30%. Find the total discounted cost of two computers and two printers that regularly cost \$899 and \$299, respectively.

Level 3: Jennifer needs a new laptop computer, color ink-jet printer, and black-and-white laser printer. She also needs a new desk and chair. Computer Depot is having a sale on all its merchandise. Computers and accessories are being discounted 25%. Furniture is being discounted 40%. Find the total cost of Jennifer’s purchase based on the prediscount prices shown below. Include a 6% sales tax on all items.

- Laptop Computer \$1199
- Color Ink-Jet Printer \$149
- B&W Laser Printer \$399

Desk	\$299
Chair	\$139

Three Levels of Geometric Problem Solving

Problem: The drawing below (Figure 1.7) consists of four circles inscribed in a square. Each circle is tangent to two other circles. The area of the square is 64 square units. Choose one of the levels below.

Level 1: Find the total area of all four circles.

Level 2: Find the area of the shaded region.

Level 3: Find the area of the hypocycloid bounded by the four circles.

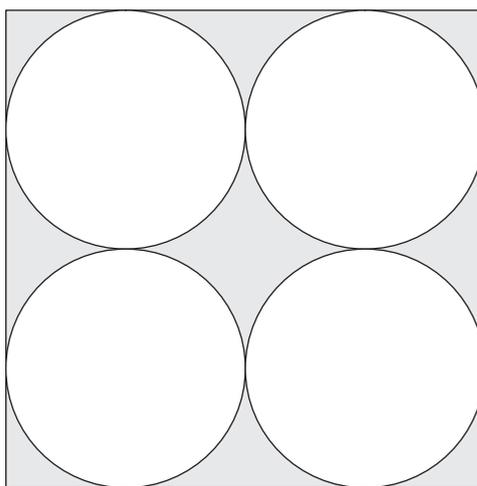


FIGURE 1.7 Hypocycloid

Why the Strategy Works

Graduated Difficulty comes from the work of Muska Mosston (1972). What Mosston discovered is that when teachers invite students into the process of analyzing and selecting the work that is most appropriate for them, the classroom dynamic changes for the better. Some of the benefits include

- Increased opportunities for all learners to succeed;
- Higher levels of student engagement and focus;
- Boosts in student confidence with more students attempting higher-level tasks;
- The development of task-analysis and self-assessment skills as students work to find the best match for themselves; and
- The establishment of a collaborative culture in which teachers work with students as they reflect on and discuss their work, their decisions, and their goals.

What makes the strategy especially appealing to students is the choice. Choice is one of the strongest, most empowering of human motivators, and classrooms that encourage decision making tend to build trust between teachers and students and build students' intrinsic motivation to learn and succeed (Erwin, 2004).

The strategy is also an ideal way to develop students' goal-setting skills. As Robert Marzano's (2007) research in *The Art and Science of Teaching* shows, encouraging students to set meaningful learning goals and helping them to evaluate and track their progress leads consistently to increased achievement levels in the classroom.

Planning Considerations

The idea behind Graduated Difficulty is that students' levels of achievement can be improved by providing *options at different levels of difficulty and challenge*. Clearly, to prepare for a Graduated Difficulty lesson, you'll need to take some time to develop the three levels of problems or tasks your students will complete.

One thing to avoid when creating levels is the temptation to base difficulty on the quantity of problems you ask students to solve. Simply asking students to solve more problems doesn't lead to the kind of task analysis and self-assessment that Graduated Difficulty naturally promotes among students. In other words, basing difficulty on the number of items will not encourage students to ask questions, such as *What skills and knowledge are needed to complete each level?* and, *How do my current skills and knowledge match up to the requirements of each task?*

Figure 1.8 on page 40 shows three ways to design levels that will promote task analysis and self-assessment on the part of students.

In addition to considering how you can challenge students with three different levels of difficulty, when designing a Graduated Difficulty lesson it is important to think about the following questions.

How will you introduce and explain the lesson? Students need to understand and be comfortable with their roles in a Graduated Difficulty lesson. Make sure students know how to analyze and select a task and reinforce the idea that it is okay for students to select whichever task is the best for them. Sara Avery, a middle school mathematics teacher, designed a poster (Figure 1.9) to help students through her Graduated Difficulty lessons.

How will students check their work? Students should be able to self-assess their work to determine if they have arrived at a correct answer and have made a good choice. To help students determine if they have completed their chosen task correctly and well, provide them with an answer key that shows how the answer was calculated. For more open-ended activities, you might choose to give students a rubric with clear guidelines and benchmarks.

How will students reflect on their learning? The ultimate goal of Graduated Difficulty is to help all students move to higher levels of thinking and learning. It is important for students to take stock of where they are

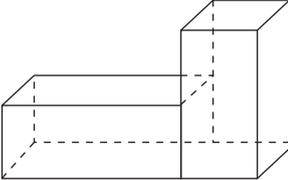
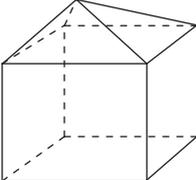
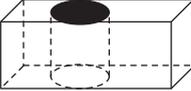
Level of	What It Means	Examples
Rigor of Content	<p>Basing levels on the degree of rigor or the depth of knowledge needed to complete each level.</p>	<p><i>Scenario 1:</i> A middle school math teacher provides three levels of equations to solve. At the first level, all equations take the form $ax + b = c$. The second level adds equations in the form $a(x + b) + c = d$, while the third level adds equations in the form $a(x + b) + c = d(x + e)$.</p> <p><i>Scenario 2:</i> While teaching geometry, a teacher might provide three levels of surface area problems. The three levels might differ according to the complexity of the figures as shown. (Each figure requires the knowledge and application of additional operations.)</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>Level 1</p>  </div> <div style="text-align: center;"> <p>Level 2</p>  </div> <div style="text-align: center;"> <p>Level 3</p>  </div> </div>
Support or Given Information Provided	<p>Basing the levels on the degree of support or background information provided in the task.</p>	<p>As part of the class's work with mathematical properties (commutative, associative, distributive, and identity), a teacher provides three levels of tasks. With each level, the amount of given information decreases.</p> <ul style="list-style-type: none"> • At Level 1, the names of the properties and a set of equations are given. Students have to identify the property or properties illustrated in each equation. • At Level 2, the names of the properties are given. Students have to create an equation illustrating each property. • At Level 3, students are given neither names nor equations. They have to create a table that includes the name of each property, an equation that illustrates it, and a simple explanation of how each equation illustrates each property.
Thinking Process	<p>Basing levels on the sophistication of thinking required by each task.</p>	<p>For a task involving data analysis (for example, two graphs with one showing the number of U.S. farms from 1950 to the present day and another showing the size in acres of the average farm over the same span), you might create your levels by</p> <ul style="list-style-type: none"> • Asking students to describe the data at Level 1 (What do the two graphs show?); • Asking students to find patterns at Level 2 (How do the two charts relate to each other mathematically? What conclusions can you make?); and • Asking students to make and explain predictions at Level 3 (What do you think the charts will show in the year 2025? How about 2100? What evidence would you use to support your predictions?).

FIGURE 1.8 How to Create Different Kinds of Graduated Tasks

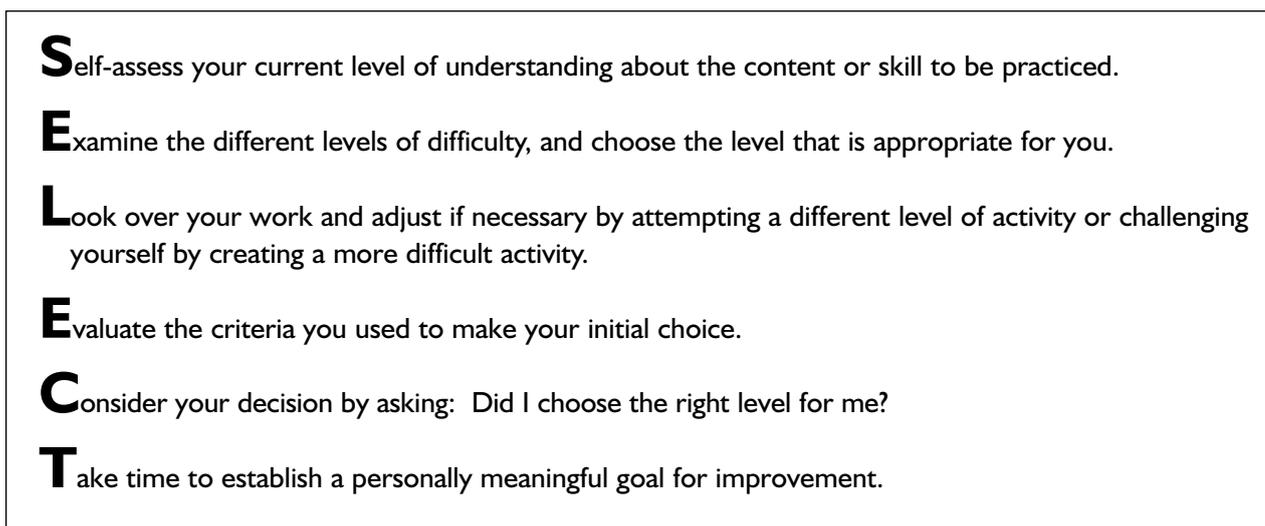


FIGURE 1.9 SELECT Graduated Difficulty Poster

at the end of the lesson so they can chart their own goals for improvement. To help students develop personal learning goals, encourage them to share their work and decision-making process with others through classroom discussion and teacher conferences. Think about how you will help students convey their thoughts and experiences with questions like these:

- What went well for you in this lesson?
- What caused you trouble?
- What criteria would you use to examine your choice and work?
- Do you think you made the right choice?
- What do you need to do to move to a higher level?

Variations and Extensions

The Graduated Difficulty strategy can be implemented with the following variations and extensions.

- Depending on the complexity of the problems students will be solving, you may choose to implement the Paired Learner model (see pages 150–158) and allow students to work in pairs. This will encourage teamwork, communication, and collaboration among students.
- The teacher can select students or pairs of students and provide them with transparencies so they can easily share their work with the class during the presentation of solutions.
- Students who have higher levels of understanding and proficiency can be encouraged and rewarded for completing all three levels of problems successfully.

New American Lecture

Strategy Overview

Mathematics is filled with content and procedures that students need to remember if they are going to succeed. For teachers, this raises a critical question: How can we make the information we present in the classroom more engaging and more memorable? New American Lecture provides a mathematics teacher with a strategic way of delivering content and providing direct instruction in a mathematical procedure. In a typical New American Lecture, the teacher provides students with four kinds—or Ps—of support:

- The teacher *prepares* students for the lecture with an engaging hook.
- The teacher *presents* brief chunks of content, which students record on a visual organizer.
- The teacher *pauses* after each chunk and poses a review question.
- The teacher provides time for students to *process* content and/or *practice* skills during and after the lecture.

How to Use the Strategy

Developing and implementing a New American Lecture requires 4 Ps:

1. Prepare a hook. Prepare a way to “hook” student interest and involve them in the content. Open discussion with a topic-related question, but with a question that may appear to be off the wall. Encourage students to think and share responses in small groups; then, open up a class discussion.
2. Present the content in chunks using a visual organizer. Chunks are subsets of the material that will either overlap or subsequently be connected to other chunks of the content.
3. Pause every 5 minutes. Pause periodically throughout instruction to ask students to think about the content in multiple ways, from different perspectives.
4. Process and/or practice the material. Assess the nature of content:
 - a. If declarative content, provide focused time for students to process the material; or
 - b. If procedural content, provide appropriate practice materials and class time.

The Strategy in Action

Alan Gorman teaches algebra. Today, Alan is using New American Lecture to teach his students about three common transformations: translations, reflections, and dilations.

Phase One: Prepare Students for Learning

Alan begins the lesson by asking students if they've seen any of the *Transformers* movies. "What do you know about Transformers? Why are they called Transformers?" Alan asks. After students offer their ideas, Alan explains that Transformers help illustrate a critical concept in mathematics: transformation.

Alan provides students with a simple definition of transformation and then helps students brainstorm some real-world examples, including

- Changing an assigned seat in the classroom;
- Changing lanes while driving;
- Changing the size of a photograph or computer graphic; and
- Changing direction while dancing.

With each new example, Alan asks students to think about two questions: One, What does each type of change look like? and two, What does each change feel like? Then, to focus students' attention on the specific content of the lecture, Alan asks students to describe, in words, the actual or apparent change in their physical position or size if they:

- Took one step backward (translation);
- Looked in a full-length mirror (reflection); and
- Viewed a 3×5 picture of themselves (dilation).

After the discussion, Alan connects students' ideas to the lesson by saying,

It turns out that our three scenarios that we've been discussing—taking a step backward, looking in a mirror, and viewing a photograph—are examples of the three most common types of transformations, which are called translation, reflection, and dilation. Of course, in mathematics, we can describe these kinds of changes in position and size with perfect precision and in more than one way. By the end of today's lesson, you will be able to describe translation, reflection, and dilation in four different ways: verbally in your own words, visually by sketching each transformation, algebraically by representing each transformation as an equation, and by identifying your own real-world example of each.

Phase Two: Present the Content

Alan distributes a visual organizer designed around the three transformations. For each transformation, students have to make notes that define the transformation, then show it visually, represent it algebraically, and cite at least one real-world example. A student's partially completed organizer looks like Figure 1.10 on page 44.

Phase Three: Pause Every 5 Minutes

Each type of transformation represents one chunk of the lecture, and each takes roughly five minutes to present. After each 5-minute segment, Alan stops lecturing, gives his students an extra minute or 2 to complete

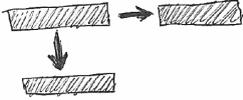
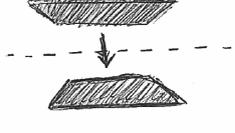
A transformation is <u>a change in position or size of a geometric figure</u>			
	Translation	Reflection	Dilation
Define	Slide a figure without turning it, preserves size and shape.	Flip a figure over a line, creating mirror image preserves size and shape.	
Visualize			
Algebraic	Given $y = f(x)$ $y_1 = f(x+h)$ or $y_2 = f(x) + k$		
Real-World Examples	sliding glass door		

FIGURE 1.10 Student's Partially Completed Organizer

their notes, and then poses a question to help students think more deeply about transformations and how to apply them. To engage all his learners and to help students develop greater perspective and understanding of the content, Alan rotates the styles of the questions he poses.

After the first chunk on translations, Alan asks a Mastery question designed to help students practice and review what they learned: *In terms of an arbitrary function $f(x)$, can you algebraically define $g(x)$, a horizontal translation of eight units?*

After the second chunk on reflections, Alan poses an Understanding question focused on comparative analysis: *Compare and contrast translations and reflections. What is similar and different about them algebraically, graphically, and numerically?*

After the third chunk on dilation, Alan poses an Interpersonal question focused on real-world applications of the three transformations: *Many careers, especially those involving design, use transformations as part of the planning and creative process. Think of a career activity that might use these three types of transformations (e.g., architect, artist, graphic designer, etc.). Describe or illustrate how all three types of transformations might be part of the career activity.*

Finally, after all of the transformation types have been presented, Alan poses a Self-Expressive question focused on visualizing and applying the three transformations: *Choose one of our unit's vocabulary words and print the letters of that word in block form on graph paper. On the same or on separate graphs, translate, dilate, and reflect the vocabulary word. Be sure to label each transformation and describe its changes from your original.*

Phase Four: Practice/Process

For the final phase of the New American Lecture, Alan wants to help students develop mastery over the skill of graphing and describing transformations. So, he provides a set of practice activities including the activity shown below.

1. Using the function $f(x) = y = |x + 2|$,
 - a. Graph, label, and describe the transformation $y_1 = f(x + 3)$
 - b. Graph, label, and describe the transformation $y_2 = f(-x)$
 - c. Graph, label, and describe the transformation $y_3 = 2f(x)$

But Alan knows that “following the procedures” is not enough when it comes to understanding mathematics deeply. So, after students have completed the activities, he presents them with a task that requires them to explain how transformations work numerically and to use their explanations to make mathematical predictions: *For each type of transformation, explain how knowing the value of the constant, in each transformation’s algebraic form, enables you to predict the pattern of change that will be seen in a corresponding set of numerical data. Use a sketch to support your explanation.*

Why the Strategy Works

The effectiveness of the New American Lecture is tied directly to the four Ps of support that it provides to students.

First, the teacher *prepares* students for the lecture using a hook. A good hook primes the engine for deep learning by engaging students’ curiosity and activating students’ prior knowledge. There are four different kinds, or styles, of hooks.

1. Mastery hooks ask students to recall information.
2. Understanding hooks ask students to use logic and reasoning to analyze an issue or controversy.
3. Self-Expressive hooks ask students to call on their imagination.
4. Interpersonal hooks ask students to draw on their personal experiences.

Examples of each of these styles of hooks are shown in Figure 1.13 (on page 49). To connect the students’ ideas that emerge in response to the hook, the teacher then bridges those responses to the content of the lecture with a simple statement that sounds like this: “These are some wonderful ideas you’ve generated. Now let’s see how your ideas relate to _____.”

For the second P of support, the teacher *presents* the content of the lecture using a visual organizer. This P actually contains two distinct supports. First, content is presented in brief three to five minute chunks. Chunking information by breaking it into manageable pieces facilitates processing and increases the likelihood of moving that information into long-term

memories where it can be recalled when needed (Gobet, et al., 2001). The second support contained within this P is the visual organizer. The visual organizer shows how all the chunks in the lecture fit together to form a whole. And as Hyerle (2000) has noted, organizers also foster an evolution in students' thinking processes: First, they learn how to manage information, and then they learn how to actively construct new knowledge.

The third P of support, *pause* every 3 to 5 minutes to pose a review question, gives students the opportunity to play with, refine, and shore up gaps in their learning. As with chunking, review questions deepen processing and help students turn the new information into long-term memories. To help students examine the content deeply and from multiple perspectives, you should rotate the kinds of questions you ask during your lectures. Figure 1.12 (page 48) shows you how to use learning styles to design and pose different kinds of questions.

The final P serves double duty. If the lecture is more declarative in nature, (e.g., famous number patterns) students will need an opportunity to further *process* the content by putting their learning to work by completing a task or creating a product. If the lecture is more procedural or skill based (e.g., how to solve problems using slope-intercept form), students will need the opportunity to *practice* their new skills.

Planning Considerations

Identify Your Topic

In mathematics, it is usually easy to select a topic, but you need to collect and chunk all of the information that you anticipate students will be learning. As you think of the topic, jot down all of the words that come into your mind. Be sure to include words, processes, or theorems that connect this topic to past mathematics.

Once you have all of the information, fit it all together. Create information chunks, or subgroups of information, by identifying the key words within the topic. These chunks will help you create a visual organizer and suggest the logical and important instructional breaks in your lecture.

Design the Visual Organizer

A good visual organizer will help students see the whole as the sum of its parts, how small bits of information come together to form a larger picture. Given your topic and its information chunks, you can design a visual organizer that both guides you and supports your students' learning during the lecture. A visual organizer should highlight your topic's key conceptual patterns and help scaffold learning. Figure 1.11 shows a variety of organizers highlighting common mathematical conceptual patterns.

Develop Review Questions

Information chunks provide opportunities for students to deal with manageable amounts of content. Plan to present these chunks in roughly

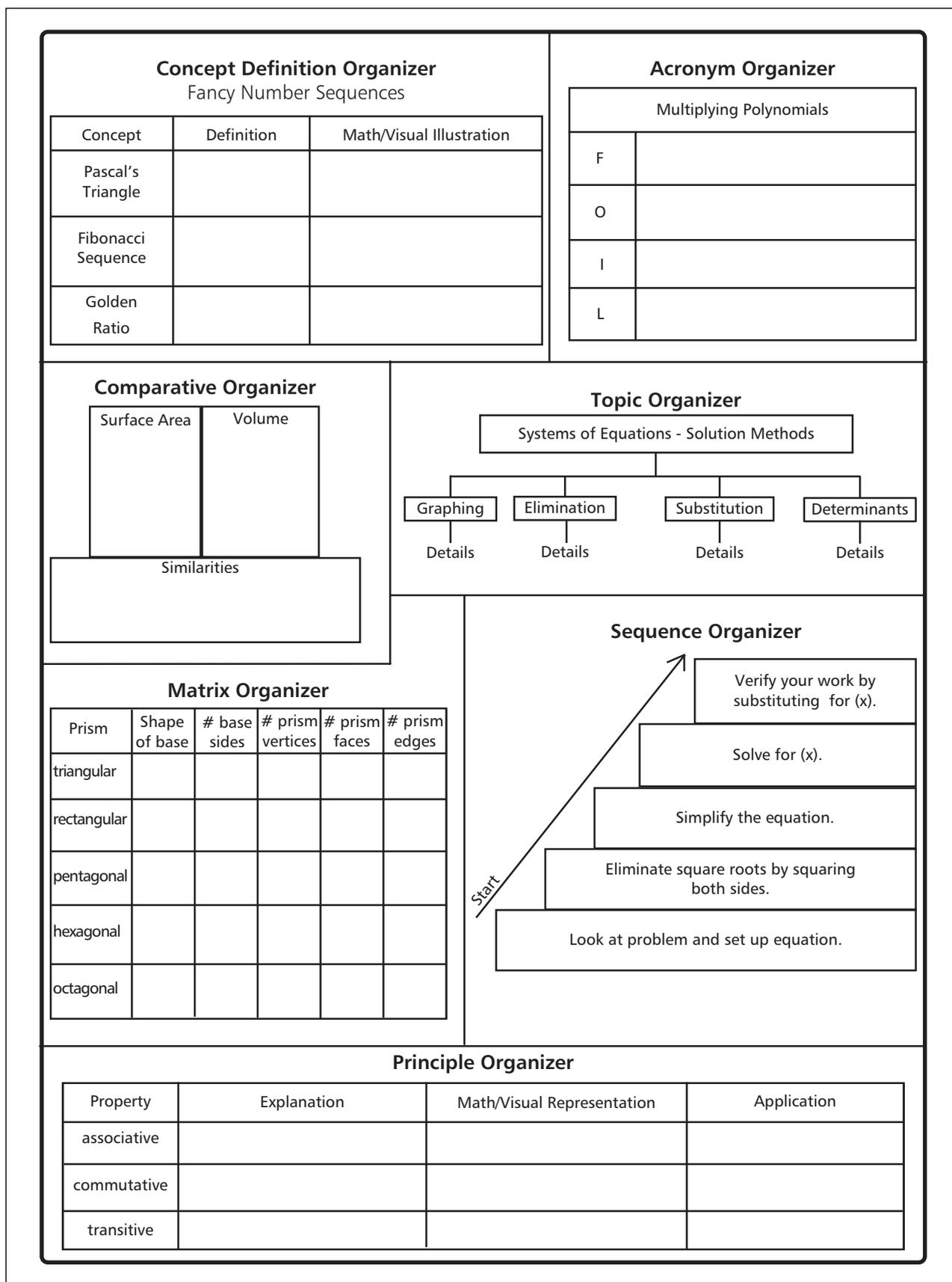


FIGURE 1.11 A Potpourri of Visual Organizers for Mathematics

5-minute intervals of lecture. Plan to stop after each lecture chunk to pose questions that will facilitate processing and require students to recall and use what they just learned. Strive to use a variety of question types during the lecture to stimulate different ways of thinking about the content. Learning styles represent one of the most effective and manageable models for incorporating a variety of questions into your teaching. Figure 1.12 below shows how the four styles of questions can be adapted to mathematical content.

Design the Hook

A hook is a designed question or activity that attracts student interest, focuses thinking, and opens memory banks closely associated with the new topic. Sometimes, a hook may appear to have little to do with mathematics. To create a hook, think deeply about your topic and the words that you first

<p>Mastery questions ask students to</p> <p><i>Recall and Practice</i></p> <ul style="list-style-type: none"> • Can you remember the steps in the procedure for solving proportion problems? <p><i>Restate</i></p> <ul style="list-style-type: none"> • Cover your organizer. How much can you remember about the commutative property? Restate it. <p><i>Summarize</i></p> <ul style="list-style-type: none"> • What is a line of best fit? Summarize in your own words. 	<p>Interpersonal questions ask students to</p> <p><i>Make Real-World Connections</i></p> <ul style="list-style-type: none"> • Can you think of three activities or situations in which accuracy of measurement is critical? <p><i>Personalize Learning</i></p> <ul style="list-style-type: none"> • What information from this lecture seems most difficult to you? Why? <p><i>Make Value-Based Decisions</i></p> <ul style="list-style-type: none"> • Which form of graphing do you like best? Why is it your favorite?
<p>Understanding questions ask students to</p> <p><i>Compare and Contrast</i></p> <ul style="list-style-type: none"> • What are the key similarities and differences between prime and composite numbers? <p><i>Prove or Disprove</i></p> <ul style="list-style-type: none"> • Argue for or against this statement: The United States should convert to the metric system. <p><i>Explain</i></p> <ul style="list-style-type: none"> • Why can't we ever put a zero in the denominator? 	<p>Self-Expressive questions ask students to</p> <p><i>Explore Metaphors</i></p> <ul style="list-style-type: none"> • How is measuring circumference like running on a track? <p><i>Use Visuals and Symbols</i></p> <ul style="list-style-type: none"> • Create an icon or sketch that represents surface area and another that represents volume. <p><i>Ask "What If?"</i></p> <ul style="list-style-type: none"> • What if there were no such thing as order of operations? What would happen?

FIGURE 1.12 Four Styles of Review Questions

Source: Adapted from Thoughtful Education Press (2007). *Questioning Styles and Strategies: How to Use Questions to Engage and Motivate Different Styles of Learners.*

<p style="text-align: center;">Mastery Hook (Focuses on Remembering)</p> <p>Take one minute to think about everything you know about the metric system. What do you already know about this system and how it's used?</p> <p>Bridge: Good! You know quite a bit about the metric system. Now, let's build some new information on top of what you already know.</p>	<p style="text-align: center;">Interpersonal Hook (Focuses on Personal Experiences)</p> <p>Have you ever found the English system difficult to work with (e.g., measuring small things, following a recipe)? What happened? Can you describe the difficulty you experienced?</p> <p>Bridge: Great! You've actually described some of the reasons that led to the development of the metric system. Let's investigate further to see what else we can learn.</p>
<p style="text-align: center;">Understanding Hook (Focuses on Reasoning)</p> <p>We use the English system in the United States, but our doctors and scientists use the metric system. Why do you think this is the case?</p> <p>Bridge: You have some good ideas! Now, let's see which of our ideas are true by investigating some of the advantages of the metric system.</p>	<p style="text-align: center;">Self-Expressive Hook (Focuses on Imaginative Thinking)</p> <p>What if the United States converted to the metric system tomorrow? How might your life change?</p> <p>Bridge: Super! You've generated a list of ways life might be more difficult. Now, let's look at some of the ways life might become easier.</p>

FIGURE 1.13 Four Hooks and Bridges for a Lesson on the Metric System

generated. What mathematics concept is embedded within? How does this concept show itself in the real world, in the student's world? Does it appear in the music, art, communication, or other interests of your students? As with your review questions, you may want to use learning styles to create different kinds of hooks and promote different styles of thinking for different lectures. Figure 1.13 shows four different ways of beginning a lecture on the metric system. Notice how each style of hook gets students to think about the content to come in a different way. Which one would you choose for your classroom?

Variations and Extensions

Visual Organizers

Visual organizers are much more than supplemental forms for students to make notes and collect ideas. Visual organizers are great learning tools that, if designed thoughtfully, can shape a lesson, give structure to a difficult reading or word problem, and reinforce important ideas and connections for students that might otherwise be lost. The educational benefits of visual organizers have been widely reported by David Hyerle (2000) and others. Teach students how to use and create simple visual organizers that will serve them in the mathematics classroom. Some of the best and most common organizers for the study of mathematics were discussed earlier in Planning Considerations (see Figure 1.11 on page 47).