

# INTRODUCTION

## MINING THE GAP

The pursuit of natural resources and precious gems is complex and hazardous. Yet, countless men and women pursue the work because the reward is so great. These individuals rely on investigation and research to determine when and where to strike. They make use of practical tools and innovation. They anticipate what will happen and plan accordingly. They are careful of unstable footing and faulty supports or bracing.

In some ways, we might think of teaching and learning mathematics similarly. Our students' success is precious to us. Achieving it is complex, and some days it even feels hazardous. We too make use of investigation and research to identify when and what to teach. Gaps in student understanding can create considerable consequences. Mathematics tasks are the tools that enable us to uncover those gaps. And high-quality mathematics tasks provide even greater insight into the quality, depth, and complexion of our students' understanding. Misconceptions and seemingly correct answers can undermine our stable footing and consistent progress.

Essentially, we must mine the gaps in our students' understanding so that we can achieve the goals we set for them. We must select and implement high-quality mathematics tasks to uncover student reasoning. We must anticipate what might happen, why it might happen, and what we will do about it when it happens.

## ABOUT THIS BOOK

Our mathematics instruction must be vibrant and engaging. We must go beyond direct instruction of rules and procedures. We must make use of much more than abundant practice and tedious worksheets. We can realize this vision with high-quality tasks that promote reasoning and representation.

Students have their own ideas about mathematics. Like their fingerprints, their ideas are unique to them. Sometimes, their reasoning is sophisticated, and other times, their reasoning is faulty. Their mistakes are not random. Instead, their mistakes are grounded in incomplete understanding,

skewed observations, or flawed logic. We must uncover their confusion so that we can make informed decisions about our instructional next steps, and high-quality tasks enable us to do so.

This book is a collection of high-quality tasks aligned to the big ideas of intermediate mathematics. It shares perspective about what we might anticipate before our students work with specific tasks. It uncovers misconceptions, incomplete understanding, and unique student perspectives through multiple student work samples for each task. It offers what we might do next for the student samples. Three additional tasks are provided for each topic. The result is 180 examples of student work and 180 quality tasks for classroom use.

All of the tasks are provided electronically for use in the classroom. Options for modifying the tasks are provided so that any task can be used in any intermediate classroom.

### WHO IS THIS BOOK FOR?

This book is designed for any stakeholder. Classroom teachers can use the tasks in this book for instruction or assessment. Student work samples can be reviewed to better understand what might happen in the classroom as well as what might be done next.

Mathematics coaches and curriculum specialists can use this book to support the design of instruction or assessment resources. It can serve as a core resource to develop professional learning around characteristics of high-quality tasks, anticipating student thinking, identifying student misconceptions, and planning instructional next steps. This book can be a centerpiece for a school-wide professional learning community, an after-school workshop series, or collaborative planning meetings.

Principals and school administrators may also use the book to guide collaborative reviews of student work by grade-level teams. Administrators may rely on it to guide the development of common assessments for benchmarking student growth. Teacher preparation programs might use it for secondary methods or diagnosis and intervention courses.

### WHAT IS THE PURPOSE? WHAT IS THE PROBLEM?

Teaching middle school mathematics is a complicated endeavor. As teachers, we must understand the concepts, procedures, and application of seemingly “basic” mathematics. We must understand and apply research-informed pedagogy. We must also understand mathematical misconceptions and apparently correct answers as well as the “logic” behind these answers.

Misconceptions and incomplete mathematical thinking may go unnoticed because we are trained to think about mathematics in black and white or correct and incorrect. As teachers, we find incorrect answers and often react with steps to reteach the correct processes. However, student misconceptions are not random. Often, we may overlook why or how the answer was generated, and so the underlying problem persists. To complicate matters, even our students’ correct answers do not always indicate accurate reasoning. For example, a student might correctly compare  $\frac{3}{4}$  and  $\frac{1}{2}$ .

Yet, that student might select  $\frac{3}{4}$  because both digits are larger rather than reason about the meaning of fractions and comparison.

As teachers, our lack of training for diagnosis is only part of the challenge. Teacher access to high-quality tasks that provide rich insight into student thinking can be limited. It can be a challenge to identify these tasks and even more challenging to create them. Yet, such tasks are necessary because of another complication with diagnosing student thinking. Unlike the perception of black-and-white answers in mathematics, student thinking and reasoning are highly variant. Simply, mistakes and misunderstandings do not always occur in the same ways or for the same reasons. They can be simplistic or complex. They can be independent of or connected to other mathematical skills and concepts.

There are two long-term ramifications of limited insight into student understanding. First and foremost, unrecognized student misconceptions become unchanged misconceptions. These misconceptions can become permanent ways of thinking. Each new layer of mathematics knowledge is then built on flawed foundations. These students are then likely to develop other misconceptions and forced to rely on rules and procedures that are lost over time. The other ramification is that the pattern continues in this teacher's classroom year after year, affecting large numbers of students.

#### HOW DOES THIS BOOK SOLVE A PROBLEM?

As teachers, we need easy access to practical, quality tasks that uncover student thinking. We need multiple examples of tasks to support instruction throughout the year. We also need help thinking about how tasks are selected, what students might do, and what to do with student responses.

Low-level tasks featuring simple recall, procedures, or algorithms are often found in textbooks. Often, these tasks yield little more than correct or incorrect answers. They provide limited insight into our students' thinking. These shortcomings can challenge us to make informed instructional decisions. Moreover, inaccurate perceptions of our students also lead to instructional missteps. Student progress and long-term retention of skills and concepts are affected.

Conversely, the rich tasks featured in this book can be used for instruction or formative assessment. They will provide opportunities for teachers to go deeper with student performance by

- considering what students do and don't know about the content,
- describing misconceptions and limited understanding through incorrect and correct responses,
- anticipating what might happen with a specific task, and
- identifying possible instructional next steps.

As a result of using these books, teachers will be able to

- identify and select rich tasks for instruction or assessment,
- consider what counts as mathematical understanding,
- anticipate and plan for student misconceptions,

- make instructional decisions based on specific misconceptions or incomplete understandings, and
- access a substantial collection of rich tasks for classroom use.

## ORGANIZATION OF THE BOOK

This book is organized around the big ideas of mathematics in Grades 6 through 8, including

- operations with fractions and decimals;
- operations with integers;
- reasoning with ratios, proportions, and percents;
- representing and manipulating expressions, equations, and functions;
- understanding key geometry concepts; and
- analyzing statistical data.

Each chapter provides

- a collection of tasks aligned to the subtopics of the big ideas,
- a brief description of each task and its importance,
- ideas about what we might anticipate our students will do with the task,
- samples of student work with descriptions of what they did with the task,
- considerations for next steps with the highlighted student work,
- three additional tasks aligned to the mathematics topic, and
- ideas about what students might do with these additional tasks.

## THE APPROACH TO STUDENT WORK

This book is a guide to rich tasks, student understanding, and misconceptions. It is shaped by authentic student work and reasoning. The student work samples are from real students in real mathematics classrooms. The tasks were collected from random classrooms so that we could see what students do when working with these tasks. Tasks were not continuously distributed until just the right samples could be found. The tasks were provided after concepts had been taught, although in many cases, it had been weeks since the concept had appeared in the classroom. None of the student samples were collected within the same period as the concept was taught. Specific student samples were selected because they represent what our students frequently do or think about the mathematics.

## COMPANION WEBSITE

All of the tasks are provided electronically on the book's companion website at [resources.corwin.com/minethegap/6-8](http://resources.corwin.com/minethegap/6-8). Note that the display of some tasks has been modified to fit the book's layout, but the full and complete versions can be found online. They can be reproduced for instruction, assessment, independent practice, or possibly for homework. Electronic downloads enable the reader to modify the tasks. Suggestions for modifications are made throughout the book.

## Three Reasons You Need This Book

1. This book provides a wealth of high-quality mathematics tasks.  
Identifying high-quality mathematics tasks can be quite difficult. We can search online for hours trying to find just the right combination of rigor, relevance, and interest. Even then, we may not find what is best for our students. Writing or creating these tasks is even more difficult. This book provides 180 tasks electronically that can be modified, enhanced, or replicated for countless possibilities.
2. This book provides insight into common approaches and misconceptions students have.  
Anticipating and identifying what students might do and the misconceptions they have can be acquired with years of experience. Yet, experience alone may not provide enough insight into what might happen with the students we're teaching this year. This book provides diverse student samples for 45 tasks related to the big ideas of mathematics in Grades 6–8.
3. This book offers ideas about what we might do next.  
We know that “louder and slower” is not the solution to incorrect student thinking. Instead, we have to consider what students know and how well they understand it. We need to pinpoint where the mathematics falls apart for our students and determine what to do next. This book highlights where things go right and where they go wrong for students. It also gives ideas about next steps for reteaching, enriching, or extending our students' thinking.